Homework Assignment #1

6.4 Suppose that the production of a pound of grapes now requires 7/8 liter of wine. If none of the other input-output coefficients change, write out the new systems for the outputs.

Answer: The production of 1 pound of grapes now requires 1/2 pound of grapes, 1 laborer, and 7/8 liter of wine. The production of 1 liter of wine still requires 1/2 pound of grapes, 1 laborer, and 1/4 liter of wine. The 10 laborers still demand 1 pound of grapes and 3 liters of wine. Let w be the amount of wine produced and g the amount of grapes produced. Wine demand is $3 + 7g/8 + w/4$; grape demand is $1 + g/2 + w/2$; labor demand is $g + w$. Setting supply equal to demand, we find:

$$\begin{aligned} w &= 3 + 7g/8 + w/4 \\ g &= 1 + g/2 + w/2 \\ 10 &= g + w \end{aligned} \qquad \text{or} \qquad \begin{aligned} 3 &= 3w/4 - 7g/8 \\ 1 &= g/2 - w/2 \\ 10 &= g + w \end{aligned}$$

Note that this has no solution. The last two equations have solution $g = 6$, $w = 4$, which contradicts the first equation.

6.8 Carry out the two checks at the end of Example 6.1 (part of Example 5).

Answer: One half unit of asset one pays $(1/2, 1, 3/2)$ in states $1, 2, 3$ while one half unit of asset two pays $(3/2, 1, 1/2)$. Adding the payoffs shows that the portfolio $(1/2, 1/2)$ pays 2 in every state, meaning it is risk-free.

The second check is whether $(1/8, 1/2, 1/8)$ is a pricing system for this economy. Here there is a problem as $\sum_i p_i R_{1i} = 1/8 + 1 + 3/8 = 3/2 \neq 1$ and $\sum_i p_i R_{2i} = 3/8 + 1 + 1/8 = 3/2 \neq 1$, violating (14).

The price vector $(1/8, 1/4, 1/8)$ is a pricing system for this economy.

7.6 Consider the general IS-LM model with no fiscal policy in Chapter 6. Suppose that $M_s = M^o$; that is, the intercept of the LM-curve is 0.

i) Use substitution to solve this system for Y and r in terms of the other parameters.

ii) How does the equilibrium GNP depend on the marginal propensity to save?

iii) How does the equilibrium interest rate depend on the marginal propensity to save?

Answer:

i) Since taxes are zero in the chapter 6 model, "no fiscal policy" means that spending is also zero (the government is neither in surplus nor deficit). That, together with $M_s = M^o$, reduces the system to

$$\begin{aligned} sY + ar &= I^o \\ mY - hr &= 0. \end{aligned}$$

This is most easily solved by substitution. The second equation yields $Y = (h/m)r$, so the first equation becomes $[(sh/m) + a]r = I^o$. Thus

$$\begin{aligned} r &= \frac{mI^o}{sh + am} \\ Y &= \frac{hI^o}{sh + am}. \end{aligned}$$

ii) The marginal propensity to save is s. Computing $\partial y/\partial s$, we find that GNP is negatively related to the marginal propensity to save. This is the well-known "paradox of thrift" in the IS-LM model.

iii) Here too, we have a negative relation between the interest rate and the marginal propensity to save.

7.14 Solve the system of equations

$$\begin{aligned} -4x + 6y + 4z &= 4 \\ 2x - y + z &= 1 \end{aligned}$$

Answer: We form the augmented matrix and then row-reduce:

$$A = \begin{pmatrix} -4 & 6 & 4 & 4 \\ 2 & -1 & 1 & 1 \end{pmatrix} \xrightarrow{(1)/-4} \begin{pmatrix} 1 & -3/2 & -1 & -1 \\ 2 & -1 & 1 & 1 \end{pmatrix}$$
$$\xrightarrow{(2)-2(1)} \begin{pmatrix} 1 & -3/2 & -1 & -1 \\ 0 & 2 & 3 & 3 \end{pmatrix} \xrightarrow{(2)/2} \begin{pmatrix} 1 & -3/2 & -1 & -1 \\ 0 & 1 & 3/2 & 3/2 \end{pmatrix}$$
$$\xrightarrow{(1)+(3/2)(2)} \begin{pmatrix} 1 & 0 & 5/4 & 5/4 \\ 0 & 1 & 3/2 & 3/2 \end{pmatrix}$$

This tells us that $x = 5/4 - 5z/4$ and $y = 3/2 - 3z/2$ will solve the system for any real number z.

7.20 Compute the rank of each of the following matrices:

$$a) \begin{pmatrix} 2 & -4 \\ -1 & 2 \end{pmatrix}, \quad b) \begin{pmatrix} 2 & -4 & 2 \\ -1 & 2 & 1 \end{pmatrix}, \quad c) \begin{pmatrix} 1 & 6 & -7 & 3 \\ 1 & 9 & -6 & 4 \\ 1 & 3 & -8 & 4 \end{pmatrix},$$
$$d) \begin{pmatrix} 1 & 6 & -7 & 3 & 5 \\ 1 & 9 & -6 & 4 & 9 \\ 1 & 3 & -8 & 4 & 2 \\ 2 & 15 & -13 & 11 & 16 \end{pmatrix}, \quad e) \begin{pmatrix} 1 & 6 & -7 & 3 & 1 \\ 1 & 9 & -6 & 4 & 2 \\ 1 & 3 & -8 & 4 & 5 \end{pmatrix}.$$

Answer: We row-reduce the matrices to determine the rank. In case (a), the rank is 1, the rank of (b) is 2, the rank of (c) is 3, the rank of (d) is 3, and the rank of (e) is 3.

7.24 Show that a square matrix A is nonsingular if and only if its row echelon forms have no zeros on the diagonal.

Answer: Recall that a square matrix is nonsingular means that its rank equals the number of rows (same as the number of columns). In other words, when we row reduce an $n \times n$ matrix, we obtain a matrix with n non-zero rows. Each row must have a leading non-zero term, and they must be in columns futher right as we go down the rows. Since there are n columns, each column must have a leading non-zero entry. The only way to make that fit is that the leading non-zero entry in each row i must be in column i. That means there are no zeros on the diagonal.

Conversely, suppose a matrix has a row echelon form with no zeros on the diagonal. Then the first n rows (all rows) in that row echelon form are non-zero, meaning the rank is n. But this is another way of saying it is non-singular.