

Mathematical Economics Exam #1, September 24, 2020

Answer all four questions. You may use any reasonable shortcuts, and may refer to your notes, my notes, and the textbook. No other reference materials may be used. You may not consult any individuals or search the internet.

You must email me your answers by 6:30pm today (September 24, 2020). To insure maximum credit, **be sure to explain your answers**. Each question is worth 25 points, for a total of 100 points. The questions are not equally hard. Good luck!

1. Consider the vectors

$$\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}, \text{ and } \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}.$$

- a) Do these vectors form a basis? Explain.
 - b) If the vectors form a basis, find the corresponding dual basis. If the vectors do not form a basis, replace one of the vectors to form a basis, then find the dual basis of that.
2. Demand for good 1 is $d_1 - ap_1 - \frac{1}{2}bp_2$; demand for good 2 is $d_2 - \frac{1}{2}ap_1 - bp_2$; the supply of good i is s_i . Here a, b, d_i , and s_i are all positive, and $s_i < d_i$. Both goods are complements.
- a) What system of equations do you get when you set supply equal to demand in both markets?
 - b) What criterion must be met in order to solve for p_1 and p_2 ? Is it satisfied?
 - c) What additional conditions, if any, must be satisfied in order to get positive equilibrium prices p_i ?
3. Let \mathbf{B} be an $n \times n$ invertible matrix and define $\mathbf{A} = \mathbf{B}^T \mathbf{B}$. Define $\langle \mathbf{x}, \mathbf{y} \rangle = \mathbf{x}^T \mathbf{A} \mathbf{y}$. Is $\langle \cdot, \cdot \rangle$ an inner product on \mathbb{R}^n ?

4. Let

$$\mathbf{A} = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & 4 & 9 & 16 & 25 \\ 1 & 8 & 27 & 64 & 125 \end{pmatrix}.$$

- a) Find rank \mathbf{A} .
- b) Find a basis for $\ker \mathbf{A} = \{\mathbf{x} : \mathbf{A}\mathbf{x} = \mathbf{0}\}$.
- c) Verify that $\text{rank } \mathbf{A} + \dim \ker \mathbf{A} = 5$.