

Mathematical Economics Exam #2, October 28, 2020

Answer all four questions. You may use any reasonable shortcuts, and may refer to your notes, my notes, and the textbook. No other reference materials may be used. You may not consult any individuals or search the internet.

You must email me your answers by 6:30pm today (October 28, 2020). To insure maximum credit, **be sure to explain your answers**. Each question is worth 25 points, for a total of 100 points. The questions are not equally hard. Good luck!

1. Consider the equation $x^3 + y^2 = 0$.
 - a) At which points is y an implicit function of x ?
 - b) What is $y'(x)$ at the points in (a)?
 - c) Is there any point on the curve $x^3 + y^2 = 0$ where the Implicit Function Theorem fails to apply?
2. Consider the constant elasticity demand function $Q = 12p_1^{-1/2}p_2^{3/2}$ where Q is the demand for good 1 and p_i is the price of good $i = 1, 2$. Suppose current prices are $p_1 = 36$ and $p_2 = 9$.
 - a) What is the current demand for Q ?
 - b) Use differentials to estimate the change in demand as p_1 increases by 4 and p_2 decreases by 0.5.
 - c) Similarly, estimate the change in demand when both prices increase by 0.4.
 - d) Estimate the total demands for situations (b) and (c) and compare your estimates with the actual demands.
3. Let $f(x) = x^4 - 4x^3 + 4x^2 + 1$. Find all of the critical points and classify them (e.g. strict local max). Are there any global maxima or minima?
4. Consider the quadratic form $Q(x, y) = x^2 + 2xy + 3y^2 - 2xz + z^2$.
 - a) Is the form positive or negative definite? Semidefinite? Indefinite?
 - b) Suppose we constrain the form to the subspace $x + 3y - z = 0$. Does it have a unique maximum or minimum on this subspace?