Mathematical Economics Final, December 7, 2021

You have until 6:55 to complete this exam. Answer all five questions. **Be sure to justify your answers!** Each question is worth 20 points, for a total of 100 points. Good luck!

1. Maximize the utility function $u(x, y) = -e^{-2x} - e^{-3y}$ subject to the budget constraints $4x + y \leq 10$, $x \geq 0$, $y \geq 0$. Don't forget to consider constraint qualification and the second order conditions.
2. Find the point on the parabola defined by $y = x^2 + 1$ that is closest to $(3, 1)$.
3. Consider the following second order homogeneous differential equation with constant coefficients:

$$\ddot{y} - \dot{y} - 20y = 0$$

   a) Find and solve the characteristic equation.

   b) State the general solution.

   c) Show that the equation has a solution for any initial data $y_0 = y(0)$ and $y_1 = \dot{y}(0)$.
4. Consider the set defined by $M = \{(x, y, z) : x^2 + y^2 - z^2 = 1\}$. Is M a 2 dimensional manifold? I.e., if $(x_0, y_0, z_0) \in M$ is arbitrarily chosen, does the Implicit Function Theorem define one of the variables as a function of the other two on a neighborhood of (x_0, y_0, z_0)?
5. Maximize $u(x, y) = 2x + 3y$ subject to the constraints $2x + y \leq 10$, $x + 2y \leq 10$, $x \geq 0$, $y \geq 0$.