

Mathematical Economics Exam #I, September 23, 2021

Answer all four questions. You may use any reasonable shortcuts. To insure maximum credit, **be sure to explain your answers**. Each question is worth 25 points, for a total of 100 points. The questions are not equally hard. Good luck!

1. Consider the linear system

$$x + 2y + 3z = a$$

$$2x + 2y + 3z = b$$

$$3x + 4y + 6z = c$$

a) For what values of a, b, c does this system have a solution?

b) When the system has a solution, when is that solution unique?

2. Consider the matrix

$$\mathbf{A} = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 1 & 8 & 27 & 4 \\ 2 & 4 & 9 & 16 \end{pmatrix}$$

a) What is the rank of \mathbf{A} ?

b) Recall $\ker \mathbf{A} = \{\mathbf{x} : \mathbf{A}\mathbf{x} = \mathbf{0}\}$. What is $\dim \ker \mathbf{A}$?

c) Find a basis for $\ker \mathbf{A}$.

3. Let V be an inner product space and $\mathbf{x}, \mathbf{y} \in V$ with $\mathbf{x} \neq \mathbf{y}$. Show that

$$\mathbf{z} = \mathbf{y} - \frac{\mathbf{x} \cdot \mathbf{y}}{\|\mathbf{x}\|^2} \mathbf{x}$$

is perpendicular to \mathbf{x} .

4. Consider the following norms on \mathbb{R}^3 . The Euclidean norm $\|\mathbf{x}\|_2 = (x_1^2 + x_2^2 + x_3^2)^{1/2}$, and the sup-norm $\|\mathbf{x}\|_\infty = \max\{|x_1|, |x_2|, |x_3|\}$.

a) Find a number A so that $\|\mathbf{x}\|_2 \leq A\|\mathbf{x}\|_\infty$.

b) What do you expect A to be in \mathbb{R}^n ?