1. Consider the equation $x^4 - x^2 + y^2 = 0$.
   
   a) For what values of $(x, y)$ with $-1 \leq x \leq 1$ does this not define a regular curve?
   
   b) If the curve is regular, for which $(x, y)$ with $x \in [-1, 1]$ can we write $y$ as a function of $x$?
   
   c) Is $\{(x, y) : x^4 - x^2 + y^2 = 0\}$ a manifold?

2. For each of the following spaces $X$ and sets $S \subset X$, determine whether each subset $S$ is an open, closed, connected, and/or compact subset of $X$.
   
   a) $X = \mathbb{R}^2$ with the usual topology. $S = \{(n, m) : \text{both } n \text{ and } m \text{ are integers}\}$.
   
   b) $X = \mathbb{R}^2$ with the usual topology. $S = \{(x, y) : x^2 + y^2 \geq 1, x^2 + y^2 < 10\}$.
   
   c) $X = (0, 1) \cup (2, 3)$ with the usual topology. The set $S$ is the interval $(0, 1)$.

3. Consider the quadratic form on $\mathbb{R}^3$ defined by

   \[ Q(x) = y^2 - 2z^2 + 2xy - 2xz + 2yz. \]

   a) Is $Q$ positive definite, negative definite, or indefinite?
   
   b) Suppose we impose add the constraint $x + 2z = 0$. Is there a constrained maximum or minimum at the origin?

4. Find the first 4 terms of the Taylor expansion of $\sin x$ about $x = 0$. 