

Mathematical Economics Exam #2, October 28, 2021

Answer all four questions. You may use any reasonable shortcuts. To insure maximum credit, **be sure to explain your answers**. Each question is worth 25 points, for a total of 100 points. The questions are not equally hard. Good luck!

1. Consider the equation $x^4 - x^2 + y^2 = 0$.
 - a) For what values of (x, y) with $-1 \leq x \leq 1$ does this **not** define a regular curve?
 - b) If the curve is regular, for which (x, y) with $x \in [-1, +1]$ can we write y as a function of x ?
 - c) Is $\{(x, y) : x^4 - x^2 + y^2 = 0\}$ a manifold?
2. For each of the following spaces X and sets $S \subset X$, determine whether each subset S is an open, closed, connected, and/or compact subset of X .
 - a) $X = \mathbb{R}^2$ with the usual topology. $S = \{(n, m) : \text{both } n \text{ and } m \text{ are integers}\}$.
 - b) $X = \mathbb{R}^2$ with the usual topology. $S = \{(x, y) : x^2 + y^2 \geq 1, x^2 + y^2 < 10\}$.
 - c) $X = (0, 1) \cup (2, 3)$ with the usual topology. The set S is the interval $(0, 1)$
3. Consider the quadratic form on \mathbb{R}^3 defined by

$$Q(x) = y^2 - 2z^2 + 2xy - 2xz + 2yz.$$

- a) Is Q positive definite, negative definite, or indefinite?
 - b) Suppose we impose add the constraint $x + 2z = 0$. Is there a constrained maximum or minimum at the origin?
4. Find the first 4 terms of the Taylor expansion of $\sin x$ about $x = 0$.