Mathematical Economics Final, December 6, 2022

You have until 6:55 to complete this exam. Answer all five questions. **Be sure to justify your answers!** Each question is worth 20 points, for a total of 100 points. Good luck!

1. Let $f(x, y) = (x^2 - y^2)^2$.
   a) Find all critical points on $\mathbb{R}^2$.
   b) Which critical points are maxima? Minima?
2. Maximize $f(x, y, z) = x + y + \sqrt{z}$ subject to the constraints $x \geq 0$, $y \geq 0$, $z \geq 0$, and $x + 2y + z \leq 5$.
3. Consider the function $f(x, y, z) = e^{x^2} + \sin xyz - z^3$ on the closed ball $B = \{(x, y, z) : x^2 + y^2 + z^2 \leq 1\}$. Without doing any computations, explain why f has both a maximum and minimum on the ball B
4. Consider the quadratic form $Q(x, y, z) = x^2 + 2xy + y^2 + 2xz + z^2$.
   a) Find a matrix A that defines the quadratic form Q.
   b) Does the quadratic form Q have a maximum, minimum, or is it indefinite at $(0, 0, 0)$ under the constraint $x + 2y + z = 0$?
5. Maximize $f(x, y, z) = x + yz$ under the constraints $x^2 + y^2 + z^2 \leq 25$, $x, y, z \geq 0$.