Mathematical Economics Final, December 6, 2022

You have until 6:55 to complete this exam. Answer all five questions. **Be sure to justify your answers!** Each question is worth 20 points, for a total of 100 points. Good luck!

1. Let \( f(x, y) = (x^2 - y^2)^2 \).
   
   a) Find all critical points on \( \mathbb{R}^2 \).
   
   b) Which critical points are maxima? Minima?

2. Maximize \( f(x, y, z) = x + y + \sqrt{z} \) subject to the constraints \( x \geq 0, y \geq 0, z \geq 0 \), and \( x + 2y + z \leq 5 \).

3. Consider the function \( f(x, y, z) = e^{x^2} + \sin xyz - z^3 \) on the closed ball \( B = \{(x, y, z) : x^2 + y^2 + z^2 \leq 1\} \). Without doing any computations, explain why \( f \) has both a maximum and minimum on the ball \( B \).

4. Consider the quadratic form \( Q(x, y, z) = x^2 + 2xy + y^2 + 2xz + z^2 \).
   
   a) Find a matrix \( A \) that defines the quadratic form \( Q \).
   
   b) Does the quadratic form \( Q \) have a maximum, minimum, or is it indefinite at \((0, 0, 0)\) under the constraint \( x + 2y + z = 0 \)?

5. Maximize \( f(x, y, z) = x + yz \) under the constraints \( x^2 + y^2 + z^2 \leq 25 \), \( x, y, z \geq 0 \).