

Mathematical Economics Exam #I, September 22, 2022

Answer all four questions. You may use any reasonable shortcuts. To insure maximum credit, **be sure to explain your answers**. Each question is worth 25 points, for a total of 100 points. You have until 6:15 to complete the exam. The questions are not equally hard. Good luck!

1. Consider the matrix

$$\begin{pmatrix} 3 & -1 \\ -1 & 3 \end{pmatrix}$$

- Find all real numbers λ where $\mathbf{A} - \lambda\mathbf{I}$ is singular.
 - For each λ found in (a), find a non-zero vector \mathbf{b} with $(\mathbf{A} - \lambda\mathbf{I})\mathbf{b} = \mathbf{0}$.
 - Do the vectors found in part (b) form a basis for \mathbb{R}^2 ?
2. Consider the following norms on \mathbb{R}^3 . The ℓ^3 norm $\|\mathbf{x}\|_3 = (|x_1|^3 + |x_2|^3 + |x_3|^3)^{1/3}$, and the sup-norm $\|\mathbf{x}\|_\infty = \max\{|x_1|, |x_2|, |x_3|\}$. (corrected)

Show that these norms are equivalent on \mathbb{R}^3 by finding positive numbers A and B with $A\|\mathbf{x}\|_3 \leq \|\mathbf{x}\|_\infty \leq B\|\mathbf{x}\|_3$.

3. Consider the matrix

$$\mathbf{A} = \begin{pmatrix} 1 & 5 & 2 & 3 & 4 \\ 1 & 8 & 4 & 9 & 12 \end{pmatrix}$$

- Find the reduced row-echelon form of \mathbf{A}
 - Recall $\ker \mathbf{A} = \{\mathbf{x} : \mathbf{A}\mathbf{x} = \mathbf{0}\}$. What is $\dim \ker \mathbf{A}$?
 - Find a basis for $\ker \mathbf{A}$.
4. Find all vectors in \mathbb{R}^4 that are perpendicular to

$$\mathbf{x}_1 = \begin{pmatrix} 1 \\ 1 \\ 1 \\ 0 \end{pmatrix} \quad \text{and} \quad \mathbf{x}_2 = \begin{pmatrix} 3 \\ 1/3 \\ 0 \\ 1 \end{pmatrix}.$$