Micro I Midterm, March 1, 2016

1. Suppose a firm uses two inputs z_1 and z_2. The production function is $f(z) = 2z_1 + 3z_2$. The net output vector is $y = (-z_1, -z_2, q)$ where where $q \le f(z)$ and $z \ge 0$. The price vector is (w_1, w_2, p).

   a) For what values of (w_1, w_2, p) is it possible to maximize profit?

      Answer: For this CRS technology, profit can only be maximized if prices are in the dual cone of the production set. Free disposal then implies $(w_1, w_2, p) \ge 0$ when profit can be maximized. Maximum profit will occur when $q = f(z)$. Then profit is $pq - w_1 z_1 - w_2 z_2 = (2p - w_1)z_1 + (3p - w_2)z_2$. Since $z \ge 0$, profit maximization is only possible if $2p - w_1 \le 0$ and $3p - w_2 \le 0$. Thus profit can be maximized if and only if $p \le \min\{w_1/2, w_2/3\}$.

   b) For what values of (w_1, w_2, p) will the supply correspondence contain only 0?

      Answer: Using the expression for profit, we find that $z = 0$ is the only profit-maximizing input if and only if $p < \min\{w_1/2, w_2/3\}$. Then $q \le 0$, so $p > 0$ is also required to insure $q = 0$. The condition is $0 < p < \min\{w_1/2, w_2/3\}$.

   c) For what values of (w_1, w_2, p) will the supply correspondence contain vectors other than 0?

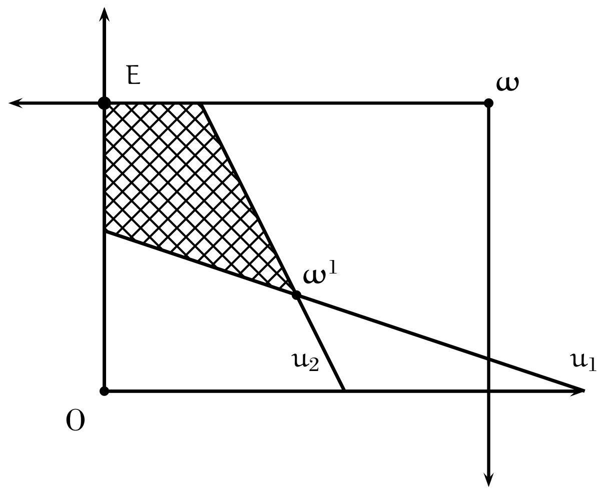
      Answer: If $p = 0$ and $0 \le \min\{w_1/2, w_2/3\}$, vectors of the form $(0, 0, q)$ for $q < 0$ will be in the supply correspondence. If $p > 0$ and $p = w_1/2$, vectors with $q = z_1 > 0$ will be in the supply correspondence, while if $p = w_2/3$, vectors with $q = z_2 > 0$ will be in the supply correspondence.

2. Consider a two-person exchange economy with endowments $\omega^1 = (2, 1)$ and $\omega^2 = (2, 2)$. The utility functions are linear: $u_1(x^1) = x_1^1 + 3x_2^1$ and $u_2(x^2) = 2x_1^2 + x_2^2$. Find all Walrasian equilibria.

   Answer: There is more than one way to solve this problem. There is the brute force method of calculating the demand correspondences and using market clearing to find the equilibrium.

   A second method is to reason our way to the answer. The aggregate endowment is $(4, 3)$. Both consumers have linear indifference curves. Consumer 1's indifference curves have slope -2 while consumer 2's have slope $-1/3$. The shaded area shows the potential mutual improvements, which lies above and to the left of the endowment point. This means that the relative price of good 2 must be between $1/3$ and 2 (the absolute slopes of the indifference curves through ω^1).

   If the relative price $p = p_1/p_2$ is greater than $1/3$, consumer one will be at a corner solution and consume only good 2 on the left boundary. If the relative price is less that 2, consumer two will be at a corner consuming only good 1 on the upper boundary. Note that the budget line with $p = 1/3$ (same as u_1) does not intersect the upper boundary inside the Edgeworth box and the budget line with $p = 2$ (u_2) does not intersect the side boundary in the Edgeworth box. This rules



out equilibria with $p = 1/3$ or $p = 2$. The only point that is both on the upper boundary and the left boundary is the corner point $E = (0, 3)$, and that is the equilibrium allocation.

The budget line must pass through both ω^1 and E, so the relative price is $p_1/p_2 = (3-1)/(4-2) = 1$. This give equilibrium price vector $\mathbf{p} = (1, 1)$ (and any scalar multiple). Consumer 1 has income 3 while consumer 2 has income 4. Consumer one can just afford $(3, 0)$ and consumer two can just afford $(0, 4)$, which is the equilibrium allocation.

3. Suppose $e(\mathbf{p}, \bar{u}) = p_1 + 2p_2 + 4\bar{u}\sqrt{p_1 p_2}$ is a consumer's expenditure function.

   a) Find the indirect utility function $v(\mathbf{p}, m)$.

   Answer: By duality, $e\big(\mathbf{p}, v(\mathbf{p}, m)\big) = m$. Thus $p_1 + 2p_2 + 4v(\mathbf{p}, m)\sqrt{p_1 p_2} = m$. It follows that $v(\mathbf{p}, inc) = (m - p_1 - 2p_2)/4\sqrt{p_1 p_2}$.

   b) Find the utility function $u(\mathbf{x})$.

   Answer: We know that the conjugate of e will be the indicator of $\{\mathbf{x} : u(\mathbf{x}) \geq \bar{u}\}$. Now consider $\mathbf{p}{\cdot}\mathbf{x} - e(\mathbf{p}, \bar{u}) = p_1(x_1 - 1) + p_2(x_2 - 2) - 4\bar{u}\sqrt{p_1 p_2}$. If $x_1 < 1$ or $x_2 < 2$ the infimum is $-\infty$, so suppose $x_1 \geq 1$ and $x_2 \geq 2$. Now consider $\mathbf{p}{\cdot}\mathbf{x} - e(\mathbf{p}, \bar{u}) = \big(\sqrt{p_1(x_1 - 1)} - \sqrt{p_2(x_2 - 2)}\big)^2 + 2\sqrt{p_1 p_2 (x_1 - 1)(x_2 - 2)} - 4\bar{u}\sqrt{p_1 p_2}$. This will have minimum $-\infty$ if $\sqrt{(x_1 - x)(x_2 - 2)} < 2\bar{u}$ and minimum 0 if $\sqrt{(x_1 - 1)(x_2 - 2)} \geq 2\bar{u}$. It follows that utility has the Stone-Geary form $u(\mathbf{x}) = \frac{1}{2}\sqrt{(x_1 - 1)(x_2 - 2)}$ for $x_1 \geq 1$ and $x_2 \geq 2$.

   Another method is to compute the Hicksian demands $h_1 = 1 + 2\bar{u}\sqrt{p_2/p_1}$ and $h_2 = 2 + 2\bar{u}\sqrt{p_2/p_1}$. Then $(h_1 - 1)/2\bar{u} = \sqrt{p_2/p_1}$ and $(h_2 - 2)/2\bar{u} = \sqrt{p_1/p_2}$. Multiplying, we obtain $(h_1 - 1)(h_2 - 2)/4\bar{u}^2 = 1$ which again yields the Stone-Geary utility $u(\mathbf{x}) = \frac{1}{2}\sqrt{(x_1 - 1)(x_2 - 2)}$.

4. Suppose utility is $u(x_1, x_2, x_3) = x_1^2 + 2x_2x_3 + 2x_1x_2 + x_3^2$.

a) For what partitions of $\{1, 2, 3\}$ is u weakly separable?

Answer: Since u is increasing, it is weakly separable with respect to $\mathcal{P} = \{\{1\}, \{2\}, \{3\}\}$. For other partitions, we compute the marginal rates of substitution: $MRS_{12} = (x_1 + x_2)/(x_1 + x_3)$, $MRS_{13} = (x_1 + x_2)/(x_2 + x_3)$, and $MRS_{23} = (x_1 + x_3)/(x_2 + x_3)$. In each case, the MRS depends on all three variables. This shows that u is not weakly separable on any partition other than $\mathcal{P} = \{\{1\}, \{2\}, \{3\}\}$.

b) For what partitions of $\{1, 2, 3\}$ is u strongly separable?

Answer: By (b), u is only separable on $\mathcal{P} = \{\{1\}, \{2\}, \{3\}\}$. That means it is not strongly separable on any partition.

c) Is u completely separable?

Answer: No, since it is not strongly separable on any partition other than ⋒P $=$ $\{\{1\}, \{2\}, \{3\}\}$.