Homework #2

9.1.3

a) Suppose a utility function is homogeneous of degree 1. Show that the Marshallian demand has an income elasticity of 1.

b) Does the same result hold when utility is merely homothetic? Explain.

Answer:

a) Corollary 4.3.6 applies, yielding $\mathbf{x}(\mathbf{p}, m) = m\mathbf{x}(\mathbf{p}, 1)$. Then $\partial x_\ell / \partial m = x_\ell(\mathbf{p}, 1)$, and the income elasticity is $\big(m/x_\ell(\mathbf{p}, m)\big)x_\ell(\mathbf{p}, 1) = 1$.

b) Yes as Corollary 2.1 applies whenever utility is homothetic.

9.3.1 Let u be a homogeneous of degree one utility function on $\mathbb{R}^L_+$. Does indirect utility have the Gorman form?

Answer: Yes, it has the Gorman form. Since u is homothetic, $\mathbf{x}(\mathbf{p}, m) = m\mathbf{x}(\mathbf{p}, 1)$. Now $v(\mathbf{p}, m) = u\big(\mathbf{x}(\mathbf{p}, m)\big) = u\big(m\mathbf{x}(\mathbf{p}, 1)\big) = mu\big(\mathbf{x}(\mathbf{p}, 1)\big)$. Setting $b(\mathbf{p}) = u\big(\mathbf{x}(\mathbf{p}, 1)\big)$, we find $v(\mathbf{p}, m) = mb(\mathbf{p})$, which is in Gorman form with $a(\mathbf{p}) = 0$.

9.3.2 Suppose utility has the Cobb-Douglas form $u(x) = \sum_k \gamma_k \ln x_k$ where each $\gamma_k > 0$ and $\sum_k \gamma_k = 1$. Show that indirect utility is additive separable.

Answer: The Cobb-Douglas demands are $x_k = \gamma_k m / p_k$. Then indirect utility is

$$v(\mathbf{p}, m) = \left(\sum_k \gamma_k \ln \gamma_k\right) + \sum_k \gamma_k \ln\left(\frac{m}{p_k}\right),$$

which is also in Cobb-Douglas form, albeit with a different constant term.

To have the same constant term would require $\sum_k \gamma_k \ln \gamma_k = 0$, or in other words $\prod_k \gamma_k^{\gamma_k} = 1$, which cannot happen as all γ_k obey $0 < \gamma_k < 1$.

10.2.1 Suppose that a utility function u obeys $v = \varphi \circ u$ where φ is an increasing function and v is homogeneous of degree one.

a) Use equation 10.2.2 to write the equivalent variation in terms of utility changes and the old price.

b) Use equation 10.2.3 to write the equivalent variation in terms of the new utility and both prices.

c) Repeat parts (a) and (b) for the compensating variation.

d) How do the compensating and equivalent variations compare when utility is homothetic?

Answer: Since v is homogeneous of degree one, we can write the expenditure function as $e(\mathbf{p}, m) = \varphi(u)\bar{e}(\mathbf{p})$ where u is the value of the indirect utility function for u at $(\mathbf{p}, m)$ and $\bar{e}$ is a function solely of prices. Then

a) $EV(\mathbf{p}^0, \mathbf{p}^1; m) = [\varphi(u^1) - \varphi(u^0)]\bar{e}(\mathbf{p}^0)$.

b) $EV(\mathbf{p}^0, \mathbf{p}^1; m) = \varphi(u^1)[\bar{e}(\mathbf{p}^0) - \bar{e}(\mathbf{p}^1)]$.

c) For the compensating variation, we have $CV(\mathbf{p}^0, \mathbf{p}^1; m) = [\varphi(u^1) - \varphi(u^0)]\bar{e}(\mathbf{p}^1)$ and $CV(\mathbf{p}^0, \mathbf{p}^1; m) = \varphi(u^0)[\bar{e}(\mathbf{p}^0) - \bar{e}(\mathbf{p}^1)]$.

d) We can use the forms $EV = [\varphi(u^1) - \varphi(u^0)]\bar{e}(\mathbf{p}^0)$ and $CV = [\varphi(u^1) - \varphi(u^0)]\bar{e}(\mathbf{p}^1)$. Then $EV, CV > 0$ if and only if $\varphi(u^1) - \varphi(u^0) > 0$. Using the other forms, we obtain $EV, CV > 0$ if and only if $\bar{e}(\mathbf{p}^0) > \bar{e}(\mathbf{p}^1)$. From this, we see that either both EV and CV are positive, or both are negative.

Thus if $EV, CV > 0$, $\bar{e}(\mathbf{p}^0) > \bar{e}(\mathbf{p}^1)$. We can then use the first form (as in part a) to see that $EV > CV > 0$. When $EV, CV < 0$, $\bar{e}(\mathbf{p}^0) < \bar{e}(\mathbf{p}^1)$. But then $\varphi(u^1) - \varphi(u^0) < 0$, so $0 > EV > CV$.