COMPARING HEDGE RATIO METHODOLOGIES FOR FIXED-INCOME INVESTMENTS

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ABSTRACT

Regression and duration are competing hedging models for reducing the risk of a debt position. This paper compares these models to determine if one method provides consistently superior hedging results. Both perfect forecast (in-sample) and historical (out-of-sample) hedge ratios are employed to hedge the long-term Bellwether bond and the two-year T-note. The regression procedure provides smaller dollar errors for the Bellwether series, but neither method is consistently superior when two-year T-notes are hedged. Comparison against a no-hedge position and two naive hedge ratio methods shows the overall superiority of the regression and duration models. Previous claims that duration is superior when end-of-period prices are known or that regression and duration should provide equivalent results are questionable.

I. THE ISSUES

Risk minimization techniques for hedging cash debt positions with futures contracts attempt to equalize the volatilities of the cash and futures positions so that the net changes in portfolio values are as close to zero as possible. Regression and duration are the two common techniques used to minimize risk for fixed income instruments. Regression employs historical data to calculate the relative volatilities of the cash and futures used for the hedge ratio, while the duration method employs the relative durations of the cash bond and futures contract to determine the hedge ratio.[1]

The main purpose of this paper is to compare the traditional regression and duration hedging models for debt instruments to determine if one method is consistently superior to the other. Duration advocates claim that when the end-of-period prices are known, then duration is a superior hedging method. However, Toevs and Jacob (1986) state that the regression and duration models are equivalent if the horizon of the hedge is instantaneous and regression uses forecasted values. The results of this paper casts doubt on the validity of both of these statements. This paper also provides

[1] A few of the pioneer empirical studies which employed the regression procedure are Ederington (1979), Figlewski (1985), Hegde (1982), Hill and Schneeweis (1982), and Kuberek and Pefley (1983). Early duration studies include Gay, Kolb and Chiang (1983) and Landes, Stoffels and Seifert (1985).

updated hedging results for long-term fixed-income instruments. More importantly, the comparison of the regression and duration models fills a gap in the current duration and hedging effectiveness literature.

The on-the-run Bellwether T-bond and the two-year T-note are employed to compare the regression and duration procedures over a 17 ½ year time span for quarterly hedging periods. Ex-post and ex-ante measures of regression and duration hedge ratios are examined, as well as comparing these risk-minimization models to a no-hedge position and two naive hedge ratio models. The results show that the regression procedure is a superior model for the Bellwether bond, while neither model is consistently superior for the two-year T-note series. Moreover, both of these methods generally have smaller variances of errors than the naive 1-1 and the naive maturity hedge procedures.

II. REGRESSION AND DURATION MODELS

A. The Minimum Variance Hedge Ratio

Ederington (1979) and Johnson (1960) employ portfolio theory to derive the minimum variance hedge ratio (HR) as the "average relationship between the changes in the cash price and the changes in the futures price" which minimizes the net price change risk, where net price change risk is measured by the variance of the price changes of the hedged position. The minimum variance hedge ratio is calculated as:

$$b^{*} = HR_{R} = \rho_{C,F}\ \sigma_{C}/\sigma_{F} \qquad (1)$$

$b^{*} = HR_{R}$ = the regression calculated minimum risk hedge ratio

σ_{C} and $\sigma_{F} = \sigma(\Delta P_{C})$ and $\sigma(\Delta P_{F})$ = the standard deviations of the cash and futures price changes, respectively

$\rho_{C,F} = \rho(\Delta P_{C}, \Delta P_{F})$ = the correlation between the cash and futures price changes.

Implementation of the regression procedure requires historical data to determine the hedge ratio, which is then applied to a future time period. However, most empirical studies of the regression method derive a hedge ratio for period t *using period t data*, which assumes that hedge ratios are stable over time. This paper uses both the coincident (perfect forecast or in-sample) hedge ratio (period t) as well as the lagged (historical or out-of-sample) hedge ratio (the period t hedge ratio applied to period t+1 data) to examine the usefulness of the regression method. Previous studies find hedging effectiveness (R^2) values at or above 79% for T-bond positions, while lower hedging effectiveness values exist for T-note positions that are hedged with T-bond futures.

Enhancements to the regression hedging procedure have appeared in the literature. A popular adjustment to the traditional regression approach is to consider the convergence of the cash and futures price to determine the hedge ratio. Castelino (1990), Herbst, Kare, and Marshall (1993), Leistikow (1993), and Viswanath (1993) examine such procedures. These convergence hedge methods provide similar hedging effectiveness values compared to the traditional regression method.[2] Ghosh (1993a, 1993b) and Ghosh and Clayton (1996) develop and use an error correction model for hedging. In this type of model cointegration is employed to integrate the long-run equilibrium relationship and the short-run dynamics of the prices. When the two price series are non-stationary but a linear combination of the series are stationary, then they are cointegrated. The existence of cointegrated series suggests that one employs an error correction model. However, the empirical results for this model also are similar to those from a traditional regression model.

Another approach is to develop a risk-return hedge ratio such as Howard and D'Antonio (1984) and Cecchetti, Cumby and Figlewski (1988). However, these methods are highly sensitive to non-stationarity in the return component when one wishes to apply historical parameters to future time periods. Cecchetti, Cumby and Figlewski (1988) and Kroner and Sultan (1993) provide time-varying ARCH models to determine the hedge ratios. However, as Kroner (1993) notes, these models are

[2] Herbst, Kare, and Marshall compare a convergence model to regression with price levels, which provides results that are not comparable to the traditional regression model. Castelino uses Eurodollar futures for hedging, which has small price changes and short maturities, making those result incomparable to hedging T-bonds. Leistikow does not empirical test his model, which includes cost of carry and information components.

highly unstable, require frequent costly rebalancing, and do not allow statistical testing. Myers (1991) shows that empirical ARCH models are no better than simpler regression models. Finally, Falkenstein and Hanweck (1996) develop a multi-futures weighted regression method in an attempt to use the information from two or more points on the yield curve for the hedge. However, they do not compare this method to the typical regression method to see whether the weighted regression procedure is superior. Overall, there is a tradeoff between using one of the unproven but mathematically elegant methods noted versus the less costly traditional regression procedure. Here we choose the less costly alternative to provide a benchmark against the traditional duration model.

B. The Duration Hedge Ratio

The duration-based hedge ratio minimizes the net price change in the value of the bond:

$$HR_D = \frac{D_C\ P_C\ (1 + i_F)}{D_F\ P_F\ (1 + i_C)} \tag{2}$$

D_C and D_F = the Macaulay durations of the cash and futures instruments

P_C and P_F = the prices of the cash and futures instruments

i_C and i_F = the yields to maturity associated with the cash and futures instruments.

The hedge ratio in (2) employs the durations of the cash and futures instruments in order to determine their relative volatilities. Empirical studies of duration find that duration reduces the unhedged risk by 73%. However, no study compares the duration and regression methods.

Kolb and Chiang (1981) indicate that the application of the duration-based hedge ratio given in (2) requires future *expected* values for the input variables as of the *termination* date of the hedge. Toevs and Jacob (1984) qualify this to state that anticipatory hedges should use expected values, while a short hedge for a currently held asset should use the current values for the cash instrument and the expected values for the duration of the futures instrument based on the (expected) delivery date. The use of expected values in the duration model is associated with the cash flows of the relevant instrument when the cash instrument is actually held, which eliminates the effect of convergence on the results. This paper uses future values in the calculation of the *perfect forecast* hedge ratios and current values for the *historical* hedge ratios.[3]

The Macaulay duration model assumes that interest rate behavior is described by a flat yield curve with small parallel shifts in the term structure. More sophisticated multi-factor duration models examined by Bierwag, Kaufman, and Toevs (1983) show limited benefits over the traditional models for estimating actual price change. Hence, the Macaulay duration model is employed here.

III. THE DATA AND METHODOLOGY

A. Inputs

Quarterly periods from 1979 through 1996 are employed in the analysis, providing a total of 71 quarters of hedge results.[4] The regression hedge ratios use weekly spot and futures price changes for each quarter in the sample to determine the appropriate hedge ratios. Both "perfect forecast" (in-sample) and "historical" (out-of-sample) hedge ratios are used to determine the per period dollar error from the hedge. The perfect forecast regression hedge ratio occurs when the hedge ratio calculated from period t is employed to hedge the price changes in period t (the conventional practice). The more realistic historical regression hedge ratios are determined by applying the hedge ratio calculated in time period t to the price changes in time t+1. Duration "perfect forecast" hedge ratios are determined by averaging the cash and futures durations at the beginning and end of time period t before calculating the hedge ratio in order to obtain average durations; this procedure minimizes the effect of a change in the duration on the results. The historical duration hedge ratio employs the durations at the beginning of the time period being analyzed.

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[3] In practice, hedgers typically use the current values of the input variables due to the difficulty in forecasting the values of these variables.

[4] Quarterly periods were chosen in order to maximize the number of periods available for analysis and because quarterly time horizons are typical for many hedgers (especially banks). While six months of data (26 weeks) could be used to generate the hedge ratios to be applied to three months of data, the overlap in input data would make the results interdependent.

Cash positions for both the Bellwether ("on-the-run") bond series and two-year T-notes are each separately hedged with the nearby T-bond futures contract. The Bellwether bond series, the most recently issued long-maturity bond series sold by the Treasury, has a significant degree of liquidity due to the volume of trading by dealers. Moreover, these bonds are hedged in large quantities by dealers and generate large price changes for given changes in interest rates. The Bellwether bond price changes typically have a high correlation with the futures price changes, usually over .95. Thus, the Bellwether bond was chosen for its liquidity, hedging activity, large price changes, and because its characteristics are similar to those of the T-bond futures contract. The two-year T-note series was chosen because its duration (characteristics) are significantly different from the T-bond futures contract; therefore, changes in the shape of the yield curve should create unstable hedge ratios for this series. Hence, the purpose of employing the two-year series is to see which method best handles the difficulties created by this type of a cross-hedge.[5]

Prices from the last day of the week, typically Friday, are used to generate the weekly price changes. Price information is obtained from The Wall Street Journal, Knight-Ridder Financial Services, and Datastream. The quarterly periods for the futures expirations end on the last week before the expiration month of the T-bond futures in order to avoid complications due to the delivery options of the futures contract. Using the first deferred futures for the delivery month provides almost identical results to the nearby futures contract. Ask prices for the cash T-bonds and T-notes are employed in the analysis, since the ask is more representative of an actual trade than is the bid.[6]

B. Methodology

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[5] The purpose of using the two-year T-notes is to determine which method deals best with a cross-hedge, not to optimally hedge the T-note. If we wanted to optimally hedge the T-note then we could use the two-year T-note futures contract, although the two-year T-note futures did not exist for a good part of the time period covered by this study.

[6] Timing differences between the cash instruments and the T-bond futures should be minimal, since the cash bonds and notes are quoted as of mid-afternoon and the T-bond futures close at 3 p.m. Eastern time. Moreover, both methods use the same data and this article concentrates on which method is superior; thus, both methods would be affected by any timing differences.

In order to compare the hedging accuracy of the regression and duration approaches we assume that an owner of $10 million (current value, not par value) of the Bellwether T-bond (and two year T-notes) wants to create a short futures hedge to protect that investment over the next three months. The results for an anticipatory hedge are simply the negative of the short hedge, therefore the existence of a *profit or loss* for the net hedged position is not the issue in evaluating the hedging results. Rather, the size of the *hedging error* is the important factor in evaluating the superiority of the hedging procedure.

The objective of hedging is to minimize the values of the average and standard deviation measures of the dollar error. A small mean dollar error shows that positive (negative) errors in one quarter are offset by negative (positive) errors in other quarters. However, small errors in *each* quarter are the objective of a good hedging procedure. Therefore, a small standard deviation of the dollar errors is a more important indicator of the ability of a given method to minimize risk. The mean absolute error also is a relevant measure, since it calculates the average error without regard for sign, as well as reducing the effect of large individual quarterly errors impounded in the squared terms of the standard deviation.[7]

IV. RESULTS

A. The Bellwether Bond

Panel A of Table 1 shows the perfect forecast (R_T and D_T) and lagged (R_{T-1} and D_{T-1}) hedge ratios for the Bellwether bond for both the regression and duration methods. The perfect forecast hedge ratios calculated in period t are applied to the period t price changes. The lagged hedge ratios are calculated in period t and employed in period t+1.[8] The average perfect forecast regression hedge

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[7] While the standard deviation finds the variability around the mean of the distribution, calculation of the deviation about an ideal value of zero provides results within $2,000 of the standard deviation about the distribution's mean. Given the more common usage of the normal standard deviation, these results are presented here and used for statistical significance tests.

[8] The lagged duration hedge ratios are determined at the beginning of period t for use in period t.

ratio in Panel A is smaller than the duration hedge ratio, although the regression hedge ratios have a slightly larger standard deviation. A t-test of the difference in the hedge ratios of the two methods is significant at the 1% level. The greater volatility of the regression hedge ratios may be due to the small sample size of the period.[9]

[SEE TABLE 1]

Panel B of Table 1 calculates the hedging effectiveness (percentage reduction in risk) of the hedged position relative to the unhedged position by using the following relationship:

$$\text{Hedging Effectiveness} = 1 - [\mathrm{var}(\Delta H)/\mathrm{var}(\Delta P_C)] \tag{3}$$

$$\Delta H = \Delta P_C - HR\,(\Delta P_F)$$

On average, regression eliminates more of the risk than does duration (93.8% to 89.7%), which is significant at the 1% level, *and* is substantially greater then the risk-reduction of other studies.[10]

Figure 1 shows the per period hedging errors for the two methods. A number of quarters have large hedging errors. While the two methods seem to possess similar errors for many of the periods, the scale of dollar errors makes the comparisons difficult. Figure 2 shows that the difference between the two methods often can be $100,000 or more. Moreover, there are periods where regression does have significantly smaller errors than the duration method. Panel A of Table 2 provides summary results for the regression and duration total dollar errors, standard deviations, and mean absolute errors per $10 million portfolios for each method and three naive approaches. The mean quarterly error, standard deviation of the errors, and mean absolute error in Panel A for the perfect forecasts

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[9] The correlation between the regression and duration hedge ratios over time is .76, indicating that there is a difference in how the two sets of hedge ratios behave over time.

[10] An alternative procedure to the hedged percentage reduction in risk compared to the unhedged position given in Table 1 Panel B is to employ the dollar error for each quarterly period, as follows:

Dollar % Reduction in Risk = 1 - | Dollar Error due to HR | / | Dollar Error due to Unhedged Position |

Using this method shows that regression provides slightly higher percentage reduction values than does duration (72% to 69%), with a smaller variability in these numbers. However, using this procedure creates eleven quarters (for both regression and duration) where the dollar percentage reduction in risk is greater than 100% (these were $120,000 or smaller errors, which were the least volatile periods in the sample, but which cause large percentage errors). Such situations occur because only the beginning and ending prices are employed to create the errors.

(R_T and D_T) are all smaller for the regression method as compared to the duration method. These results imply that when perfect information forecasts of the hedge ratios (i.e. information concerning future volatility) for long-term bonds is available, then the regression method is superior to duration for hedging purposes. Both the regression-based and duration-based mean quarterly errors increase when historical information (R_{T-1} and D_{T-1}) is employed. Standard deviations and mean absolute values of the dollar errors also increase, although not substantially. However, overall, the historical lagged regression hedge ratios still provide a *smaller* mean error, standard deviation, and mean absolute error than the duration method when the Bellwether bond series is employed. Panel A of Table 2 also provides the results for an unhedged position, a 1-1 naive hedge ratio, and a naive maturity-based hedge ratio. The regression and duration methods do a very credible job of reducing the risk of the unhedged position. Moreover, the more sophisticated methods are superior to the naive methods in terms of standard errors and mean absolute error.

[SEE FIGURES 1 AND 2 AND TABLE 2]

Panel B of Table 2 determines the percentage of the number of periods where one method is superior to the others. The first table in Panel B shows that the regression method has a smaller error than any of the other methods (including duration) for 58% to 82% of the quarterly periods. Duration is superior to the unhedged and maturity hedged methods but is not superior to the 1-1 method. The second table in Panel B shows the statistical significance of this binomial method for the number of superior periods; the statistical test employed is the matched pairs sign test. The null hypothesis is that there is no significant difference between the proportion of times one method is superior to another method (the probability p* = 50%). Thus,

$$\mu_p = p^* \tag{4}$$

$$\sigma_p = \sqrt{p^* q^* / n} \tag{5}$$

$q^* = 1 - p^*$

n = the number of observations

and $$z = (p - \mu_p)/\sigma_p \tag{6}$$

with z being the standardized normal variate. The results show that regression is significantly better than the duration and naive methods for all comparisons. Duration is superior to the no hedge and maturity hedge methods, but there is no significant difference between the duration and 1-1 hedge methods.

Panel C of Table 2 shows the results from using a t-test to evaluate the difference between the standard deviations of the errors from the various methods. A t-test is employed rather than an F-test since there is a significant correlation between the error series being compared. The t-test is calculated as:

$$t = \frac{(\sigma_a^2 - \sigma_b^2)\,(\sqrt{(n-2)})/2}{\sigma_a\,\sigma_b\,\sqrt{(1 - \rho_{ab}^2)}} \tag{7}$$

with a, b referring to the two series

ρ_{ab} = correlation between series a and b

The results in Panel C show no statistical difference between the regression method and duration procedures, but both techniques are superior to the naive methods.

Table 2 Panel D shows the results for testing the significance of the differences between methods for the mean absolute errors. The statistical test is a paired two sample t-test, where each quarter for one method is paired with the same quarter for the second method.[11] The results for the Bellwether bond shows that the regression method is superior when forecasted values are employed but there is no significant difference when historical values are used. Both methods are vastly superior to the no hedge and maturity hedge but neither historical method is significantly different from the 1-1 naive hedge procedure.

Overall, one can conclude that for the Bellwether bond series (which has characteristics similar to the T-bond futures contract) the regression model is superior to duration, while both of these

[11] This test does *not* assume equal variances.

methods are superior to the naive and no-hedge strategies. However, these results may be influenced by the volatility structure of the data. Therefore, the next section examines the T-bond results in more detail by separating the data into different types of volatility.

B. Further Analysis of the T-bond Hedges

A more in-depth look at the T-bond results provides some interesting information. Figures 1 and 2 suggested a change in the volatility and error structure of the hedges in 1987. In fact, breaking the data into two equal intervals as of the fourth quarter of 1987 separates the data into a more volatile first half and a less volatile second half.[12] Table 3 Panel A shows the average and standard deviation of the hedge ratios for the two time intervals. It is evident (which is confirmed by the t-values which test the differences between the periods) that the hedge ratios for the first half of the data are significantly higher than for the second half, for both the regression and duration results. The more volatile first half has larger hedge ratios for each procedure. Panel B shows that the regression method is superior to the duration method in the first half (for both the perfect forecast and historical measures), while there is no significant difference between the methods in the second half of the data. These conclusions are confirmed by the same statistical tests that were performed in Table 2 (not shown here).[13] Also note that the dollar errors decline significantly from the first to the second half of the data for both models. Moreover, both the regression and duration methods are superior to the no-hedge and naive hedge methods in the first half of the period, but there is no difference between these methods and the naive 1-1 hedge in the second half of the period.

[SEE TABLE 3]

Given that the differences reported in Table 3 are associated with volatility, a closer look at the

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[12] The size and volatility of the quarterly price changes, the size of the dollar hedging errors, and the difference between the regression and duration errors all confirm the appropriateness of this splitting of the data.

[13] The regression method is superior to duration for 69% and 61% of the quarters in the first half, for the perfect forecast and historical data, respectively. In the second half of the data regression has smaller errors for an (insignificant) 46% and 54% of the quarters.

individual volatile quarters is appropriate. Seven of the thirteen quarters with dollar errors over $200,000 for regression are associated with large price changes in the cash T-bond; nine out of fourteen quarters for duration have large price changes.[14] Since each one point represents a $100,000 change in the cash price, inaccurate hedging can have a large effect on the errors. However, those quarters with large errors can *not* be associated with large changes in their hedge ratios. On the other hand, an interaction between the following factors could have an effect when large price changes occur: (1) the effect of large price changes on the hedge ratios of the methods due to outliers for regression and convexity effects for duration; (2) the dollar errors are based on only two prices (the beginning and end of the period), while the hedge ratio for regression is calculated from weekly data and the duration hedge ratio is based on the characteristics of the bond and initial interest rate; (3) timing differences in the cash and futures price (although minimal in general, they could be important during volatile times). Overall, the quarters with large price changes are often associated with large errors, but a number of the large errors do not have large price changes. Hence, the size of the price change is not the only factor affecting the results.

Finally, we undertake an examination of the time series behavior of the errors. Figure 1 seems to show a negative serial correlation in the dollar errors. However, the correlation in the errors for the regression model is +.28 and for the duration model is +.31. On the other hand, the changes in the hedge ratio for the regression model are *negatively* correlated (-.38) while the duration hedge ratio changes have a correlation of +.19. While the dollar error serial correlations are significant, they explain only a small proportion of the variability of the results, and the dollar errors do not have a distinguishable pattern with the hedge ratios (moreover, the correlation in price changes is an insignificant -.04).

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[14] For regression, four of the quarters had price changes of more than nine points and three had price changes of five to nine points from the beginning to the end of the quarter; two additional quarters had price changes of three to five points. For duration, six quarters had price changes greater than nine points, one with seven to nine points, four with five to seven points, and one with three to five points.

C. Two Year T-notes

Table 4 Panel A provides the perfect forecast and (lagged) historical hedge ratios for the two-year T-note hedges for the regression and duration models. As with the Bellwether series, Panel A shows that the regression hedge ratios are smaller on average than the duration hedge ratios, but the regression hedge ratios vary more.[15] The hedge ratios are statistically different from one another. Panel B of Table 4 shows that the average reduction in risk for regression is larger than for duration (53.3% to 27.9%).[16]

[SEE TABLE 4]

One might expect the dollar errors for the two-year T-note hedges to be smaller than the errors for the T-bond series, since the price changes for two-year T-notes are much smaller than for T-bonds. However, Figure 3 (and Table 5) show that the cross-hedge of T-notes with T-bond futures causes the T-note hedge errors to be comparable in size to those for the Bellwether bond. Figure 3 shows that the errors for regression are larger than those for duration during most of the first half of the series. However, for the latter half of the series regression provides smaller errors than does duration. Figure 4 shows that the two methods can give substantially different errors for the same quarter. Table 5 Panel A shows that the regression method is inferior to the duration series for both the perfect forecast and historical results for all three measures of the dollar error values, although the differences are not large in most cases. The historical regression results have a substantially larger standard deviation than the perfect forecast hedge ratios, while duration shows no comparable increase. However, the mean absolute error has only a small change for both methods. Also, the absolute errors are almost identical for the two methods for the historical results. Panel A also provides a comparison of the

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[15] The correlation between the regression and duration hedge ratios over time is .91, showing that the two methods are similar in how their hedge ratios vary over time.

[16] Using the dollar errors to find the percentage reduction in risk (as in footnote 10) gives a dollar risk-reduction for T-notes that is less than for the Bellwether bond, with the duration method providing superior results to regression (43% risk reduction for duration compared to 31% for regression). However, as with T-bonds, there are a number of quarters (15) where the percentage reduction in risk was greater than -100%, due to small dollar errors. These periods were omitted from the calculation of the figures in this footnote.

regression and duration results to the unhedged and naive methods. Both the duration and regression methods are clearly superior to the unhedged and naive hedging positions.[17]

[SEE FIGURES 3 AND 4 AND TABLE 5]

Table 5 also provides the statistical test results for the two-year T-note that are equivalent to those given for the Bellwether bond in Table 2. Panel B of Table 5 shows that neither regression nor duration is superior to the other in terms of the number of periods where one method has the smaller dollar error. However, both methods are superior to the unhedged and naive methods. Panel C of Table 5 tests for the significant differences in the standard deviations of the errors. The duration method possesses a smaller (statistically significant) standard deviation than given by the regression method (using both the forecasted and historical values), as well as a smaller standard deviation than the naive methods. Moreover, there is no significant difference between using the forecasted vs. historical duration values. The regression method also is superior to all of the naive methods.

Table 5 Panel D for the T-note series tests for differences in the mean absolute errors. Neither duration nor regression provides a statistically smaller error compared to the other, for either the forecasted or historical values. Both methods are superior to all of the naive hedge procedures.

Overall, for the two-year T-Note series, duration is superior to regression for one of the three statistical tests and duration has somewhat smaller dollar errors. However, the evidence is so unconvincing that neither method is deemed superior to the other. The next section examines specific characteristics of the T-note results.

D. Further Analysis of the T-note Hedges

Table 6 shows the hedge ratios and dollar errors when the T-note data is broken into two equal time periods. Similar to T-bonds, this dichotomy is a natural result of the smaller price changes,

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[17] Note that the unhedged position has a substantially smaller standard deviation and mean absolute error than does the 1-1 naive method. This shows the problem in using a 1-1 ($100,000 futures to $100,000 cash) hedge when the maturities, and hence the volatilities, differ substantially between the futures and cash positions.

volatility, and dollar errors in the second half of the data. As with T-bonds, the hedge ratios for both the regression and duration methods decline substantially from the first half to the second half of the data. Panel B of Table 6 shows that duration provides smaller errors and standard deviations than regression in the first half of the T-note data, but that the two methods are almost identical in the second half. The dollar errors dropped by two-thirds from the first half to the second half of the data. While both methods are superior to the no hedge and naive methods in the first half, there is no significant difference between these methods and the naive maturity model in the second half.[18]

One cause of large errors may be non-parallel shifts in the yield curve, which could create difficulties for both the duration and regression models. Separating out the nine largest periods where a large change in the difference between the long-term and short-term interest rates occurs, i.e. where a change in the slope of the term structure is more than a 1% change in the spread of long and short-term rates, shows such shifts are important. The measures of errors are significantly larger when a large change in the spread occurs; in particular, the absolute dollar error is $348, 473 and $284,047 for the regression and duration methods for the nine quarters with the largest spread changes, while the errors are $66,386 and $65,003 for the other quarters.[19] The reason for the large regression errors can be traced to large changes in the hedge ratio for 7 of the 9 quarters. Of course, the duration hedge ratios changed little, since the durations of the underlying T-note were almost constant, but the effect of the differences in convexity between the T-bond futures and the cash T-note obviously had a major effect during these intervals. Hence, a method to consider such changes in the slope of the yield curve could improve these results. For duration, Lee and Oh (1993) suggest a method for duration, although this method has not been tested. For regression, Falkenstein and Hanweck (1996)

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[18] Duration is superior to regression for 58% and 64% of the quarters for the first half for the perfect forecast and historical methods, respectively. In the second half, the regression method was superior 54% and 60% of the time. As with the T-bond data, the conclusions noted here are confirmed by statistical tests but are not shown here for space reasons.

[19] The dollar errors for each of the nine quarters was above $100,000 for regression, while eight of the nine quarters errors were above $100,000 for the duration method. The mean dollar errors were $166,198 and $148,440 for the nine quarters for the two methods, respectively. All measures of the error indicate that the large non-parallel shifts in interest rates had a greater effect on the regression model as compared to the duration model.

provide a weighted regression method that considers different points on the yield curve.

[SEE TABLE 6]

Examining the size of the price changes for the T-note data provides similar results to that of the T-bonds. Two of the quarters with dollar errors above $100,000 have price changes of over six points, and two more have changes of two to four points. Five quarters with large errors have changes less than two points. Hence, while the size of the price change may have an effect, it is not the dominant factor affecting the errors.

The time series correlation of the dollar errors is -.37 for regression and -.16 for duration (the opposite sign compared to the serial correlation for T-bonds). The correlation of the changes in the hedge ratios are -.55 for regression and +.36 for duration. All but the duration dollar error correlation is significant, but the serial correlation only explains less then 14% of the total variability.

V. CONCLUSIONS

Regression and duration are two hedge ratio methods used to reduce risk. This paper compares these methods to each other and to the unhedged position and two naive hedge methods. For the Bellwether bond series, the regression method is superior to all of the other methods, including duration. On the other hand, there is no significant difference between the duration and naive 1-1 hedge for the Bellwether bond series. When all of the evidence is examined, neither duration nor regression is consistently superior to the other for the two-year T-note series, although duration does tend to provide smaller errors when a large change in the slope of the yield curve occurs. Further analysis of the results shows that regression and duration can give substantially different results for specific quarters; thus, these are *not* "equivalent" techniques.

The positive results of this paper conflict with two statements made about these two techniques. Toevs and Jacob (1986) claim that the two methods are equivalent if regression uses forecasted values. Gay and Kolb (1983) state that when end-of-period prices are used then the duration method is superior. Neither statement is supported by the majority of tests in this paper. Possible extensions to this paper include comparing these results to other regression and duration models for hedging and to change the length of the hedge period.

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TABLE 1
BELLWETHER BOND HEDGE RATIOS AND HEDGING EFFECTIVENESS

Panel A: Hedge Ratios

	$HR(R_T)$	$HR(D_T)$	$HR(R_{T-1})$	$HR(D_{T-1})$
Mean	1.099	1.246	1.100	1.256
σ	0.173	0.162	0.174	0.154

t-value for difference:
$HR(R_T)$ vs. $HR(D_T)$ = -5.23*
$HR(R_{T-1})$ vs. $HR(D_{T-1})$ = -5.65*
$HR(R_T)$ vs. $HR(R_{T-1})$ = -.03
$HR(D_T)$ vs. $HR(D_{T-1})$ = -.35
* Significant at the 1% level

Panel B: Hedging Effectiveness

Method	Average % Reduction in Risk	σ of % Reduction
Regression	93.8%	6.9%
Duration	89.7%	10.9%

t-value for mean difference = 2.694*
*Significant at the 1% level

TABLE 2
EVALUATION OF BELLWETHER RESULTS

Panel A: Dollar Errors

	Error Due $HR(R_T)$	Error Due $HR(D_T)$	Error Due $HR(R_{T-1})$	Error Due $HR(D_{T-1})$	Error Due No Hedge	Error Due 1-1 Hedge	Error Due Maturity Hedge
Mean	-$48,564	-$69,110	-$63,499	-$74,972	$15,433	-$41,742	-$75,463
σ	$167,913	$183,050	$180,850	$187,139	$694,031	$218,152	$282,454
Abs. Error	$130,066	$147,660	$142,691	$153,218	$533,071	$161,177	$214,826

Panel B: Percentage of Periods that Regression/Duration is Superior to Column Variables (71 periods)

	Perfect Forecast Values				Historical Values			
Method	Duration	No Hedge	1-1	Maturity	Duration	No Hedge	1-1	Maturity
Regression	58%	82%	65%	66%	58%	77%	58%	62%
Duration		77%	52%	66%		76%	49%	63%

Matched Pairs Sign Test for Percentage of Superior Periods (t-values)

	Perfect Forecast Values				Historical Values			
Method	Duration	No Hedge	1-1	Maturity	Duration	No Hedge	1-1	Maturity
Regression	1.30[c]	5.30[a]	2.47[a]	2.71[a]	1.30[c]	4.60[a]	1.30[c]	2.00[b]
Duration		4.66[a]	0.35	2.71[a]		4.36[a]	-0.12	2.24[a]

A positive value indicates that the row variable is superior to the column variable.
All unstarred values are not significant at the 10% level
[a] Significant at the 1% level
[b] Significant at the 5% level
[c] Significant at the 10% level

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Panel C: Statistical Significance of the Difference in Standard Deviations (t-values)

	Regression (t)		Regression (t-1)			
Method	Duration (t)	Regression (t-1)	Duration (t-1)	No Hedge	1-1	Maturity
Regression	-1.06	-1.38[b]	-.46	-16.27[a]	-2.56[a]	-6.25[a]
			Duration (t-1)			
			Duration (t)	No Hedge	1-1	Maturity
Duration			-0.98	-14.55[a]	-1.62[c]	-6.97[a]

A negative value indicates that the row variable is superior to the column variable.
All unstarred values are not significant at the 10% level
[a] Significant at the 1% level
[b] Significant at the 5% level
[c] Significant at the 10% level

Panel D: Paired Two Sample t-test for Difference of the Mean Absolute Errors

Method	Duration (t)	Regression (t-1)	Duration (t-1)	No Hedge	1-1	Maturity
Regression (t)	-1.77[b]	-1.69[c]		-7.89[a]	-2.51[a]	-3.76[a]
Duration (t)			-.86	-7.30[a]	-.16	-4.12[a]
Regression (t-1)			-.99	-7.65[a]	-1.09	-3.12[a]
Duration (t-1)				-7.27[a]	.05	-4.06[a]

A negative value indicates that the row variable is superior to the column variable.
All unstarred values are not significant at the 10% level
[a] Significant at the 1% level
[b] Significant at the 5% level
[c] Significant at the 10% level

TABLE 3
T-BOND RESULTS BY SUBPERIOD

Panel A: Hedge Ratios

		$HR(R_T)$	$HR(D_T)$	$HR(R_{T-1})$	$HR(D_{T-1})$
1st half:	Mean	1.156	1.346	1.155	1.349
	σ	0.204	0.138	0.207	0.162
2nd half:	Mean	1.040	1.144	1.044	1.162
	σ	0.107	0.116	0.110	0.061
t-value of difference in HR		2.95*	6.57*	2.77*	6.33*

* Significant at 1% level

Panel B: Dollar Errors

1st Half:	Error Due $HR(R_T)$	Error Due $HR(D_T)$	Error Due $HR(R_{T-1})$	Error Due $HR(D_{T-1})$	Error Due No Hedge	Error Due 1-1 Hedge	Error Due Maturity Hedge
Mean	-\$6,725	-\$26,913	-\$19,902	-\$39,686	\$56,378	\$8,944	-\$23,863
σ	\$204,938	\$231,675	\$227,518	\$242,004	\$837,456	\$283,081	\$325,691
Abs. Error	\$149,262	\$175,581	\$160,581	\$183,943	\$632,037	\$209,429	\$230,118
2nd half:							
Mean	-\$91,598	-\$112,513	-\$107,095	-\$110,259	-\$26,683	-\$93,877	-\$128,537
σ	\$105,134	\$99,545	\$103,429	\$99,710	\$515,880	\$99,850	\$222,127
Abs. Error	\$110,321	\$118,942	\$120,214	\$117,239	\$431,276	\$111,546	\$199,097

TABLE 4

TWO-YEAR T-NOTE HEDGE RATIOS AND HEDGING EFFECTIVENESS

Panel A: Hedge Ratios

	$HR(R_T)$	$HR(D_T)$	$HR(R_{T-1})$	$HR(D_{T-1})$
Mean	0.202	0.261	0.204	0.263
σ	0.117	0.081	0.117	0.080

t-value for difference:
$HR(R_T)$ vs. $HR(D_T)$ = -3.47*
$HR(R_{T-1})$ vs. $HR(D_{T-1})$ = -3.51*
$HR(R_T)$ vs. $HR(R_{T-1})$ = -.08
$HR(D_T)$ vs. $HR(D_{T-1})$ = -.16
* Significant at the 1% level

Panel B: Hedging Effectiveness

Method	Average % Reduction in Risk	σ of % Reduction
Regression	52.6%	28.0%
Duration	40.4%	35.5%

t-value for mean difference = 3.16*
*Significant at the 1% level

Two quarters are omitted due to the large variability in the basis for the duration method (caused by large changes in the bond futures price). The resulting “reduction in risk” value for duration is substantially greater than -100%, which would distort the results.

TABLE 5
EVALUATION OF TWO-YEAR T-NOTE RESULTS

Panel A: Dollar Errors

	Error Due $HR(R_T)$	Error Due $HR(D_T)$	Error Due $HR(R_{T-1})$	Error Due $HR(D_{T-1})$	Error Due No Hedge	Error Due 1-1 Hedge	Error Due Maturity Hedge
Mean	$13,559	$3,168	$11,604	-$3,205	$17,350	-$33,274	$11,666
σ	$169,519	$155,259	$187,477	$155,516	$262,528	$421,811	$216,575
Abs. Error	$101,707	$92,354	$98,556	$94,318	$156,499	$322,716	$124,079

Panel B: Percentage of Periods that Regression/Duration is Superior to Column Variables (71 periods)

	Perfect Forecast Values				Historical Values			
Method	Duration	No Hedge	1-1	Maturity	Duration	No Hedge	1-1	Maturity
Regression	48%	66%	82%	62%	48%	68%	77%	59%
Duration		65%	80%	59%		63%	79%	59%

Matched Pairs Sign Test for Percentage of Superior Periods (t-values)

	Perfect Forecast Values				Historical Values			
Method	Duration	No Hedge	1-1	Maturity	Duration	No Hedge	1-1	Maturity
Regression	-0.35	2.71[a]	5.30[a]	2.00[b]	-0.35	2.95[a]	4.60[a]	1.53[c]
Duration		2.47[a]	5.07[a]	1.53[c]		2.24[b]	4.83[a]	1.53[c]

A positive value indicates that the row variable is superior to the column variable.
All unstarred values are not significant at the 10% level
[a] Significant at the 1% level
[b] Significant at the 5% level
[c] Significant at the 10% level

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Panel C: Statistical Significance of the Difference in Standard Deviations (t-test)

	Regression (t)		Regression (t-1)			
Method	Duration (t)	Regression (t-1)	Duration (t-1)	No Hedge	1-1	Maturity
Regression	1.77[b]	-1.84[b]	4.60[a]	-5.26[a]	-22.07[a]	-3.54[a]
			Duration (t-1)			
			Duration (t)	No Hedge	1-1	Maturity
Duration			-.21	-7.83[a]	-10.02[a]	-6.52[a]

A negative value indicates that the row variable is superior to the column variable.
All unstarred values are not significant at the 10% level
[a] Significant at the 1% level
[b] Significant at the 5% level
[c] Significant at the 10% level

Panel D: Paired Two Sample t-test for Difference of the Mean Absolute Errors

Method	Duration (t)	Regression (t-1)	Duration (t-1)	No Hedge	1-1	Maturity
Regression (t)	1.19	.40		-4.66[a]	-6.15[a]	-2.89[a]
Duration (t)			-.75	-4.05[a]	-6.55[a]	-2.91[a]
Regression (t-1)			.52	-4.19[a]	-5.62[a]	-2.75[a]
Duration (t-1)				-3.93[a]	-6.55[a]	-2.76[a]

A negative value indicates that the row variable is superior to the column variable.
All unstarred values are not significant at the 10% level
[a] Significant at the 1% level
[b] Significant at the 5% level
[c] Significant at the 10% level

TABLE 6
T-NOTE RESULTS BY SUBPERIOD

Panel A: Hedge Ratios

		$HR(R_T)$	$HR(D_T)$	$HR(R_{T-1})$	$HR(D_{T-1})$
1st half:	Mean	0.265	0.322	0.265	0.324
	σ	0.106	0.063	0.108	0.064
2nd half:	Mean	0.137	0.197	0.142	0.201
	σ	0.090	0.035	0.092	0.031
t-value of difference in HR		5.41*	10.15*	5.10*	10.07*

* Significant at 1% level

Panel B: Dollar Errors

1st Half:	Error Due $HR(R_T)$	Error Due $HR(D_T)$	Error Due $HR(R_{T-1})$	Error Due $HR(D_{T-1})$	Error Due No Hedge	Error Due 1-1 Hedge	Error Due Maturity Hedge
Mean	$21,673	$7,827	$30,734	-$2,521	$22,603	-$14,652	$18,026
σ	$231,654	$209,937	$258,217	$211,638	$361,150	$426,436	$300,298
Abs. Error	$150,590	$130,840	$144,005	$132,095	$243,015	$305,858	$193,475
2nd half:							
Mean	$5,213	-$1,625	-$7,525	-$3,889	$11,947	-$52,427	$5,124
σ	$61,483	$64,901	$62,445	$65,501	$87,016	$422,344	$60,422
Abs. Error	$51,428	$52,769	$48,993	$52,767	$67,511	$340,056	$52,701