THE SECRET OF TUNING BY HARMONICS

by David Dolata

Tuning is perhaps one of the most overlooked areas of classical guitar performance practice.[1] Since it is done *before* the actual playing of the music begins, we tend to think of tuning as preliminary, but in reality it is an integral and critical part of the performance. The final tuning check immediately before the performance begins is most crucial to the success of the performance, yet in our impatience to begin playing, this is the tuning check that is most frequently given inadequate attention. When, as frequently happens, this inattention is compounded by ineffective tuning technique, it isn't until the first sour chord that the performer notices the problem.

The secret is that the widely-used method of tuning by comparing harmonics at the 5th and 7th frets simply *does not and cannot work* without major adjustments. This method tunes the natural harmonic at the 7th fret (VII) on the 5th string (5) to the natural harmonic at V on 6; 4 is tuned to 5, 3 to 4, and 1 to 2 in the same manner.[2] The 2nd string can be tuned by comparing it to either the natural harmonic at VII on 6 (what I will call *Harmonic Method A*) or the natural harmonic at IV on 3 (*Harmonic Method B*). These tuning methods are fast and flashy, but *cannot work because of the fundamental difference between the size of the fifth*[3] *produced at the 7th fret by natural harmonics (pure) and those produced on the guitar fingerboard (equal-tempered).*[4] A brief introduction to historical tunings and temperaments and why they developed will provide an understanding of this discrepancy and how it makes accurate tuning by harmonics (at V and VII) virtually impossible.

Some tuning problems, of course, may not be due to the method, but rather to the instrument. Before moving on, it is important to check your instrument to see if the frets on your guitar are positioned properly. Appendix I outlines several instrument-related tuning problems and their solutions.

Pythagorean Tuning vs. Equal Temperament

Temperament refers to the various manners in which the octave can be divided into twelve pitches. Unfortunately, *there is no possible way to divide the octave into twelve pitches so that they are each acoustically pure.*[5] Equal temperament is one of many temperaments that have been used over the centuries to make these twelve notes "fit" within the octave. The first major tuning system that achieved widespead use was the system believed to have been put forward by Pythagoras (570-504 B.C.).[6] Since tuning by harmonics uses Pythagorean fifths, we will spend some time comparing Pythagorean tuning to equal temperament.

Prominent in the Pythagorean system are fifths and fourths that are acoustically pure and major thirds that are very wide. There are also two sizes of semitone, diatonic (eg., E-F, F#-G) and chromatic (F-F#, Bb-B, etc.). As fifths and fourths were the predominant intervals, tuning by this method served the music well.[7] Table I below is a "cents chart" comparing equal temperament (row 1) to Pythagorean tuning (row 2). The number represents the size of the interval from E. For example, a major third from E-G# in equal temperament is 400c. and a Pythagorean major third is 8c. wider, 408c. Note that 100 cents is an equal-tempered semi-tone and 1200c. equals an octave.

TABLE I[8]

	E	F	F#	G	G#	A	Bb	B	C	C#	D	D#	E
ET:		100	200	300	400	500	600	700	800	900	1000	1100	1200
PT:		90	204	294	408	498	588	702	792	906	996	1110	1200

Table II compares the sizes of selected intervals between equal temperament and Pythagorean tuning:

TABLE II

	Equal Temperament	Pythagorean
Octave	1200c. (pure)	1200c. (pure)
Fifth	700	702 (pure)
Fourth	500	498 (pure)
Major Third	400	408
Minor Third	300	294
Diatonic Semitone	100	90
Chromatic Semitone	100	114

Figure I (see next page) is an approximate illustration of how your frets would be positioned comparing Pythagorean tuning to equal temperament. Notice that on the Pythagorean fingerboard the 5th fret is a little closer to the nut and that the 7th fret is a little farther from the nut than on the equal-tempered fingerboard. If you consult Tables I & II, you will see that the Pythagorean fourth (5th fret) is slightly narrower than the equal-tempered fourth and that the Pythagorean fifth (7th fret) is slightly wider than the equal-tempered fifth.

Figure II is not to scale, but isolates these relationships. While these differences by themselves are slight, when they are totalled the discrepancy becomes quite significant.

This is the heart of the matter. The natural harmonic at the 7th fret produces a pure (Pythagorean) 12th while the interval of a fifth (or 12th) produced anywhere on the fingerboard, including the 7th fret, is equal-tempered.[9] If the guitar's frets were positioned in a Pythagorean arrangement, using this type of harmonic tuning would work just fine, but since its frets are in equal temperament it cannot work. Let us turn our attention to the reason for this discrepancy.

Equal Temperament and the Pythagorean Comma

Equal temperament is a tuning system designed to compensate for the fact that a cycle of twelve *acoustically* pure fifths exceeds seven pure octaves by 24c. Equal temperament rectifies this discrepancy (the *Pythagorean comma*) by dividing the 24c. equally over the twelve perfect fifths. Thus each fifth is tempered (narrowed) by 2 cents, from a pure perfect fifth of 702c. to a slightly tempered fifth of 700c.[10] Figure III below is an illustration of the circle of fifths. The fifths inside the circle are equal-tempered (700c.) and the fifths outside the circle are pure (702c.). Notice that each pure fifth covers a slightly larger portion of the circle's circumference than does its respective equal-tempered fifth. Nevertheless, this amount is significant enough that by the time the progression has returned to the top of the circle, the pure fifths have overshot the original starting pitch by 24c., 8424c. rather than 8400c. (seven pure octaves). Equally-tempered fifths allow the final fifth to return to the exact starting point. Otherwise, there would be two distinctly different pitches for C (assuming the circle started at C).

Table III below shows the cents each fifth has accumulated as it progresses around the circle.

FIGURE I

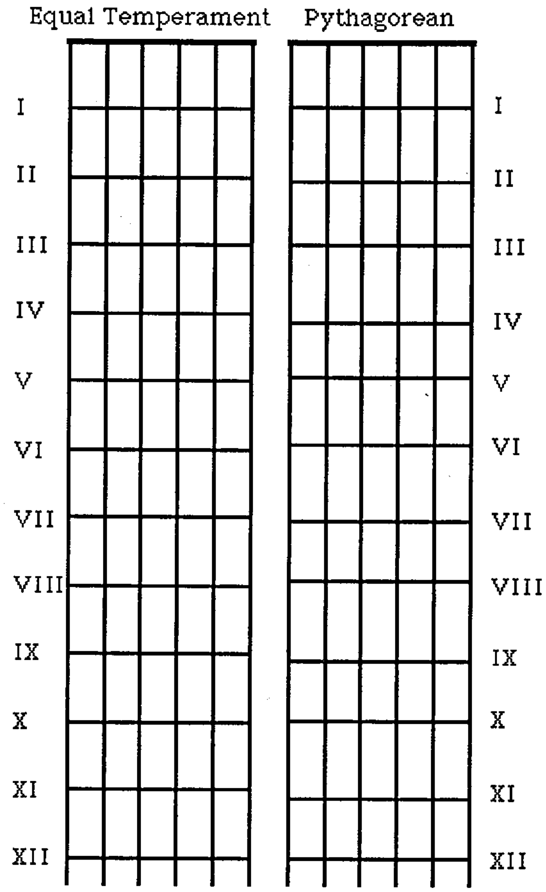

FIGURE III

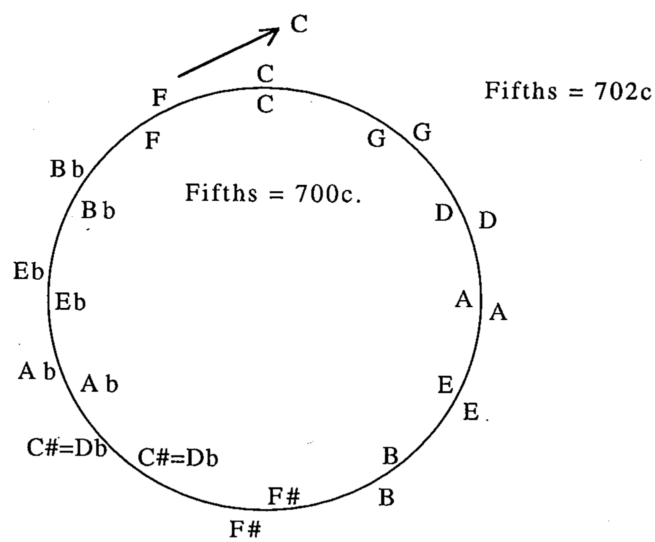

FIGURE II

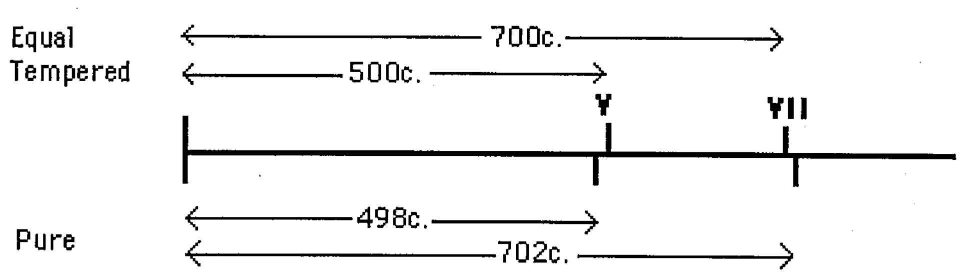

Table III

	Equally Tempered	Pure
C-G	700	702
G-D	1400	1404
D-A	2100	2106
A-E	2800	2808
E-B	3500	3510
B-F#	4200	4212
F#-C# (Db)	4900	4914
Db-Ab	5600	5616
Ab-Eb	6300	6318
Eb-Bb	7000	7020
Bb-F	7700	7722
F-C	8400	8424

8424
\- 8400
24

The problem with equal temperament is that the equally-tempered fifths result in major thirds that are noticeably higher than pure thirds (400c. rather than 386c.). This is the trade-off for a system that is equally *serviceable* in each key.[11]

The Effect of the Difference in Size Between Pure and Equal-tempered Fifths On Tuning the Guitar

With these discrepancies in mind, let us now examine the effect they have on tuning. Keep in mind that harmonics represent *pure intervals*. As indicated above, this presents no problem at the 5th fret because the octaves are pure in equal temperament, but at the 7th fret the harmonic yields a pure 12th of 1902c. rather than the equal-tempered 12th of 1900c. This same outcome is duplicated between 5 and 4, 4 and 3, and 2 and 1, each case resulting in open fourths that are narrower (and strings that are tuned higher) than in equal temperament by an accumulated total of 8c.

When tuning by harmonics, the goal is to get the natural harmonic at VII to match the natural harmonic at V on the previous string (i.e., the adjacent string on the bass side). In other words, since the natural harmonic at V on the lower string is 2400c. above the its own open string, the harmonic at VII on the highest string must also be 2400c. above the open lower string. This amount is achieved by adding the size of the higher string's 7th fret harmonic (1902c.) to the size of the interval between the open strings:

Lower String Harmonic at V		Upper String Harmonic at VII
		1902c.
	+	498c.
2400c.	=	2400c.

As you can see, when these harmonics are tuned to each other, it is inescapable that the interval between the two strings will be 498c., a pure fourth, and 2c. narrower than an equal-tempered fourth at 500c. Thus, the *upper* string is 2c. too high when compared to equal temperament. Conversely, when the interval between the open strings is an equal-tempered fourth, the *harmonic* at VII on the upper string is sharp by 2c. when compared to the lower string's harmonic at V:

1902c.
\+ 500c.
2402c.

You can have either a) an equal-tempered fourth between the open strings or b) matching harmonics at V and VII. You cannot have both.

2nd String - Harmonic Method A

There are two ways to tune the 2nd string, each resulting in a rather dramatic discrepancy. *Harmonic Method A* is more common, but also more deceptive. The open B string is matched with the harmonic at VII on 6, 1902c. above the open string. When the first string is tuned to the second string in the same manner as the other fourths, the resulting interval from the 6th string is 2400c.

1902c. (the size of the interval between 6 and 2)
\+ 498c. (the size of the interval between 1 and 2)
2400c. two pure ocataves

However, the real problem lies in the size of the interval between the 2nd and 3rd strings, 408c. This is 22c. (a *syntonic* comma)[12] higher than a pure major third, and 8c. higher than an equal-tempered major third. This amount is derived by subtracting 1494c. (which represents the three fourths already tuned—498 times 3) from the 1902c.

1902c. (6 to 2)
-1494c. (6 to 3)
408c.

The two E strings are in perfect accord since the negative difference from equal temperament caused by the tuning of the fourths [-8c.] is cancelled by the positive amount [+8c.] the open third exceeds equal temperament. This is particularly insidious because while there is no cumulative discrepancy, there is a total of 16c. discrepancy *within* the two octaves when compared to equal temperament and 22c. when compared to pure intervals.

2nd String - Harmonic Method B

Harmonic Method B tunes the 2nd string harmonic at V (B) to the 3rd string harmonic at IV, also a B.[13] This particular harmonic provides a pure major 10th (major third plus an octave) above the open string, that is, a B that is 1586c. above the G, 14c. lower than the equal-tempered third at 1600. This difference alone is very noticeable. The 1st string is then tuned to the 2nd as mentioned above. If you add the 14c. (the amount by which the interval between 3 and 2 is narrow and by which the 2nd string is sharp) to the previous 8c., the total is 22c. (the syntonic comma). This represents the accumulated amount that the 1st string is sharp and the amount the two octaves between 6 and 1 are narrow as illustrated in Table IV below. The end result is the same as if we were able to accurately tune the open strings to pure fourths and a pure third.[14] Table IV clearly illustrates that it is impossible to fit four pure 4th and a pure major third into two octaves without coming away with a shortfall. This is important to remember because harmonic tuning treats the guitar's fourths, fifths, and thirds as if they were pure, which, as we know, they are not.

Table IV

	Equal-Tempered	Pure	Difference
fourth	500c.	498c.	- 2c.
fourth	500c.	498c.	- 2c.
fourth	500c.	498c.	- 2c.
third	400c.	386c.	-14c.
fourth	500c.	498c.	- 2c.
	2400c.	2378c.	-22c.

Comparison of Harmonic Methods A & B

Table V compares Harmonic Methods A & B:

Table V[15]

String	Harmonic Method A	Harmonic Method B	Equal Temperament
6			
	498c. (+498c.)	498c. (+498c.)	500c. (+500c.)
5			
	498c. (+996c.)	498c. (+996c.)	500c. (+1000c.)
4			
	498c. (+1494c.)	498c. (+1494c.)	500c. (+1500c.)
3			
	408c. (+1902c.)	386c. (+1880c.)	400c. (+1900c.)
2			
	498c. (+2400c.)	498c. (+2378c.)	500c. (+2400c.)
1			

Comparing Harmonic Methods A & B, notice that they are identical until the 3rd string. At this point Method A places the syntonic comma (22c.) all in one place, between the 2nd and 3rd strings (408c. - 386c. = 22c.) while Method B, by continuing with pure intervals, expresses the syntonic comma as the difference between the two outer strings and two pure octaves (2400c. - 2378c. = 22c.). Table V shows that equal temperament is, in effect, a *deliberate mistuning* from pure of the fourths and the third so that first, they fit within the two octaves and second, no interval is mistuned to the degree that it is unusable, though the third is rather sharp.[16]

Table VI below lists each individual open string and the amount it is sharp or flat[17] from the actual pitch that string would be if it were tuned in equal temperament, assuming that the guitar is tuned from the 6th string to the 1st:[18] It is important to remember that this chart only measures the string in relation to its distance from the 6th string. *In relation to the 6th string*, Method A corrects itself by the time it reaches the 1st string. Method B becomes progressively worse, with any note on the 1st or 2nd strings being very noticeably flat.

To sum up what has been presented so far, tuning by either Harmonic Method A or B without adjustments simply cannot work. *The reason is that while the natural harmonic at the 7th fret is a pure 12th, the 12th (or fifth) produced anywhere else on the guitar, by virtue of the way its frets are positioned, is equal-tempered.* Individually, the discrepancy between the fifths (or 12ths) is not significant, but as they accumulate, *the discrepancy is compounded and accurate tuning is made impossible.*

Harmonic Method A—the Worst Offender[19]

The problem with Harmonic Method A, as you will recall, is not so much the relationship between distant pairs of strings, but rather the relationship between the 2nd and 3rd strings, which are 408c., a Pythagorean third and 8c. wider than in equal temperament. Table VII shows the amount in cents that any interval between each pair of strings will be sharp or flat.

Table VI

String	Harmonic Method A	Harmonic Method B
6		
5	- 2c.	- 2c.
4	- 4c.	- 4c.
3	- 6c.	- 6c.
2	+ 2c.	- 20c.
1	+/- 0c.	- 22c.

Table VII

6 - 5 = -2	5 - 2 = +4
6 - 4 = -4	5 - 1 = +2
6 - 3 = -6	4 - 3 = -2
6 - 2 = +2	4 - 2 = +6
6 - 1 = +/-0	4 - 1 = +4
5 - 4 = -2	3 - 2 = +8
5 - 3 = -4	3 - 1 = +6
	2 - 1 = -2

For example, in a five-note C Major chord, both thirds (between 5 & 4 and 1 & 2) are 2c. too narrow and the fifth (between 5 & 3) is 4c. too narrow. The 10th is 2c. too wide, both octaves are 4c. too wide, both sixths 6c. too wide, and the fourth between the 2nd and 3rd strings is 8c. too wide. The total absolute discrepancy is 38c., 8c. on the flat side and 30c. on the sharp side. In fact, the result would be the same for *any* five-note chord from the 5th to 1st strings inclusive. Six-string chords have a "dissonance factor" of 54c. and four-string chords from the 4th to 1st strings have a dissonance factor of 28c.

The Perception of Consonance in Harmonic Method A

Even though the dissonance factor will be the same for all six-string chords, other factors result in a *perception* of consonance when Method A is used. Chords that have octaves on the outside strings (E, F, G, major and minor, etc.) do not sound nearly as out-of-tune as chords that have either an interval other than an octave between 6 and 1 (e.g., a six-string C major chord - a 13th) or chords that use only four or five strings (e.g., five-string C major, A or D major or minor, etc.). The purity of the octaves to some degree masks the dissonance in the rest of the chord.[20] This is particularly treacherous because E major is the chord most guitarists use to check the accuracy of their tuning. The E chord may sound almost tolerable, but the C and D chords will sound dreadful.

Referring to Table VII, notice that there is a bias toward the flat side in the bass strings and a bias toward the sharp side in intervals involving the top two strings. Even more significant is that the dissonance factors involving pairs of strings tend to be greater where at least one of the strings (of the pair) is found on the 4th through 1st strings.[21] Furthermore, chords that have thirds (which define the major/minor quality of the chord) in the treble strings (e.g., C, A, and D major and minor) sound worse than chords where the thirds are less conspicuous.

Method A Can Be of Some Use with Adjustments

However, even though these tuning methods are inaccurate, they do have the advantage of being able to get you into the ballpark quickly. Once inside, you can find your seat by making adjustments based on your knowledge of the discrepancies inherent within these methods. Or you can use one of the methods described below. For this purpose Harmonic Method A is more useful than Method B because it places the 1st string accurately and the 2nd string closer than does Method B.[22]

At this point it is better to think in terms of how the natural harmonics sound in relation to each other once the guitar is tuned accurately. If your guitar is tuned properly, as we discovered previously, the harmonic at VII on 6 should be 2c. higher than the 2nd string open. The harmonic at VII on 5 should be 2c. higher than the harmonic at V on 6, etc. for the V to VII relationship. Begin by tuning your outer strings together by tuning V on 6 to the 1st string open. Then, if you tune your 5th, 4th, 3rd, and 1st strings' 7th fret harmonics slightly sharp as compared to their lower strings' 5th fret harmonics, you should be fairly close. The open B string should be slightly flat as compared to the 7th fret harmonic on the low E string. While these adjustments can work, I do not recommend this method because it is considerably more difficult to accurately estimate 2c. sharp or flat than it is to precisely match unisons or octaves. In the end you will end up checking by one of the other tuning methods anyway. I think it is wiser to use a more efficient method right from the start.

Recommended Tuning Methods

There is always the 5th fret-4th fret method that we all learned as beginners. It's not flashy, *but it works.* This method is made easier if it is done backwards from the way it is usually described. In other words, start from the 1st string and work your way down to the 6th. This method is better than the reverse because as mentioned above, it is easier to delineate the difference between higher pitched intervals than between lower pitched intervals. Your ear has the opportunity to warm up before you get to the more difficult task of tuning the lower strings. This method is also more professional especially when playing in a group with other instruments because the standard tuning pitch given is the A at the 5th fret of the 1st string. If we guitarists want to be considered on a par with other instrumentalists, we can't be asking for an E when everyone else uses an A.[23] Using the 5th fret to set the pitch also reinforces its use for tuning the other strings.

A very fast and accurate tuning method which can be used by itself or as a check of the 4th-5th fret method is tuning by the natural harmonics found at the 12th fret.[24] Remember that one of the problems with the harmonic tuning methods described above is the use of harmonics representing pure fifths from their open strings.[25] However, since the octave and unison are the only pure intervals in equal temperament, the harmonics at the 12th fret are safe since they represent a pure octave above the open string. The harmonic at XII on the 6th string should match the E at VII on 5 and the E at II on 4, the harmonic at XII on 5 should match the A at II on 3, the harmonic at XII on 4 should match the D at III on 2, the harmonic at XII on 3 should match the G at III on 1, and the harmonic at XII on 2 should match the B at VII on 1. This method is even faster if you use your 4th finger to get the harmonic at XII and your 1st finger for the fretted note.

From your experience you know that since our strings are tuned to each other, one slight error that is hardly noticeable is enough to ruin the overall tuning, once that error is compounded. Therefore, as a final test, check a number of octaves (non-natural harmonic) along with a few choice chords, preferably selected from the music to be played.[26]

Pre-Performance Tuning Tips

In preparation for a performance, make sure that you have checked and rechecked your tuning *before* taking the stage. In front of an audience, you are more likely to become distracted and make tuning errors. Providing that you have done a thorough and accurate tuning backstage, a very quick and quiet check should be adequate. If you are playing with other instruments, you may need to take more time to be sure that the *ensemble* is in tune. When performing with a keyboard instrument, it is imperative that you check each of your open strings with its respective note on the keyboard. The keyboard instrument may have drifted out of tune and since you can retune and they cannot, you must adjust.[27]

The most certain way to avoid the possibility of human error is to use an electronic tuner for your pre-concert tune-up. (These tuners won't become distracted by an audience.) If you do use an electronic tuner, be sure to keep a spare battery in your case. You might even consider bringing your tuner on stage with you for added insurance. If this makes you uncomfortable, use one of the methods previously described above.

Tuning is a critical aspect of classical guitar performance practice that should not be approached with nonchalance, but with focused attention. There are always things we cannot control once we take the stage, but we can at least give ourselves a head start by avoiding tuning methods that simply cannot work.

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APPENDIX I

Determining Theoretically-Correct Fret Positions

Frequently, guitars are set up inaccurately or in such a way as to make them impossible to tune properly. The distance of each fret from the nut is determined by multiplying the fretting factor by the length of the string (in centimeters or millimeters). In order to use the following factors, measure the string length of your guitar and multiply that amount by the respective factor to determine the correct theoretical placement of the fret. Compare the results to the fretting of your instrument.

Fret	Fretting Factor[28]
1.	.05612569
2.	.10910128
3.	.15910358
4.	.20629947
5.	.25084646
6.	.29289322
7.	.33258007
8.	.37003948
9.	.40539644
10.	.43876898
11.	.47026845
12.	.50000000

While these factors will give you the correct *theoretical* position, they cannot tell you the precise location of each fret since they do not take into account such variables as the shape of the neck, fret height, string diameter, density, or composition—all of which affect the exact placement of the fret. These figures also do not take into consideration the compensation luthiers make for "sharpening."[29]

Sharpening

When a string is fretted, it is stretched tighter, causing the note to be sharper than it would normally be in that position. The result is that the frets must be placed slightly closer to the nut to compensate for this sharpening effect. The amount of compensation necessary to counter this effect increases as the frets get closer to the bridge because of a) the shorter string length and/or b) the higher action.[30] This means that if your fret is set slightly closer to the nut than the fretting factor indicates, the fret is probably set correctly. If not, there are a number of possible problems. If the frets are all consistently off in one direction, it may be that your bridge is incorrectly positioned. If your guitar plays increasingly sharp as you progress up the neck, it may be that having your saddle compensated by a competetent luthier will be the solution.[31] However, if the inaccuracy seems either random or dramatic, you may need to have your frets repositioned.

False Strings

Tuning problems can also be caused by strings that are false. If a portion of the length of a string is smaller in diameter than the rest of the string, the string is false and cannot be tuned. You can tell if your string is false by comparing the natural harmonic at the 12th fret with the fretted note at the 12th fret. If the harmonic is not precisely one octave above the open string it may be false.[32]

While strings may come from the manufacturer false, the method of stringing the instrument can also make the string false. Once the string is tied to the bridge, the tendency is to grasp the string near the bridge to pull the knot tight. This concentrates all the tension from the pulling on a short section of the string. The string can be stretched so tight that its diameter over that section is decreased, making the string false in that spot. A simple way to avoid this problem is to grasp the string from the other end, beyond where the string would cross the nut. This method distributes the tension equally over the length of the string. If you do find that a string is false, you may be able to salvage the string by reversing it (i.e., tying the end that was at the head to the bridge). This only works if the false portion of the string was near the bridge (which is most likely for the reason given above) because now the section of the string that was part of the vibrating length is wrapped around the tuning peg.

Appendix II

The Size of the Major Third as a Function of the Size of the Fifth

By consulting Figure III and using the major third C-E as our example, you can see how the size of the major third is derived by adding four fifths (in cents) and subtracting two octaves from the total:[33]

	Equal Temperament	1/4 Comma Meantone	Pure (Pythagorean)
C-G	700c.	696.5c.	702c.
G-D	700c.	696.5c.	702c.
D-A	700c.	696.5c.	702c.
A-E	+ 700c.	+ 696.5c.	+ 702c.
	2800c.	2786c.	2808c.
	-2400c.	-2400c.	-2400c.
	400c.	386c. (pure)	408c.

Notice that the four pure fifths (2808c.) exceed two octaves and a pure major third (2400c. + 386c. = 2786c.) by 22c:

2808c.
-2786c.

22c.

This discrepancy is known as the *syntonic comma.* A more familiar way of thinking about it for guitarists is that the syntonic comma is the amount by which four pure fourths and a pure major third (E-A-D-G-B-E) fall short of two octaves: 2400c. - 2378c. = 22c. Like the Pythagorean comma, the syntonic comma is simply another way of expressing the mathematical and aural discrepancies inherent in our musical system.

1/4 Comma Meantone has pure major thirds because each fifth is tempered (or narrowed) by 1/4 of the syntonic comma (22c. divided by 4 = 5.5c.), resulting in fifths of 696.5c. In exchange for its pure major thirds, 1/4 Comma Meantone sacrifices serviceability in each key. In general this temperament is arranged so that the concentration of keys near the top of the Circle of Fifths sounds quite good, while the keys farther away from the top of the circle get progressively more dissonant.

1/6 Comma Meantone is a practical compromise between 1/4 Comma Meantone and Equal Temperament. It divides the 22c. by 6 (22c. divided by 6 = 3.66) resulting in fifths that are 698.34c.—not quite as flat as in 1/4 Comma. The major thirds are not quite pure, but noticeably sweeter than equal-tempered thirds. In exchange for the greater-than-pure thirds, 1/6 Comma Meantone is usable in more keys than 1/4 Comma Meantone.

Lutenists are finding that by adjusting their fret positions, they can enhance their tuning by using these historical temperaments. For further information on historical precedents for this practice see Lindley, *Lutes, viols and temperaments.*

Appendix III

An Experiment with Pure Intervals

By tuning your open strings to four acoustically pure perfect fourths and an acoustically pure major third you will find that the 1st string will be noticeably flat. Try to make your open fourths as pure as possible, with no beats whatsoever. (Pure fourths are a slight bit narrower [flatter] than equal-tempered fourths.) Now concentrate on the third between the G and B strings. See if you can delineate the limits both flat and sharp where the interval is obviously no longer a major third. Once you have found those limits, see if you can hear the varying degrees of "thirdness." You might hear versions of your third that sound gentle and others that sound harsh. Try to get the third as pristine as possible, with no beats at all. At first this third may seem rather flat compared to the equal-tempered third, but with focused listening you should be able to hear how stable it is. Now try to tune the 1st string as a pure fourth with the 2nd string just as you did with the other strings. If you did indeed tune pure fourths as well as a pure major third (no easy task with ears accustomed to equal-tempered intervals), as mentioned above, your 1st string will be noticeably flat when compared to the open 6th string. It will fall short of two perfect octaves by 22c. (the syntonic comma); 2c. for each fourth (8c.) and 14c. for the third. This experiment demonstrates how it is impossible to fit four pure fourths and a pure major third into two octaves without coming up short.

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End Notes

1 I am indebted to Ross Duffin, Paul O'Dette, and Clare Callahan for their many useful suggestions and insights regarding this topic.

2 Frequently the 1st string is tuned to an outside source (a tuning fork, another guitar, or an electronic tuner) and then the 6th string is tuned to the 1st string by comparing its 5th fret harmonic to the open 1st string. As we will see later on, this does not in itself cause a problem.

3 Actually, the harmonic at VII is a 12th, i.e., a fifth plus an octave.

4 Tuning by Harmonic Method B produces the same results as tuning the open strings pure to each other.

5 That is, so that there is no beating whatsoever. Beating refers to rhythmic pulsations (alternating increases and decreases in volume) that are heard when an interval is nearly, but not quite in tune.

6 To learn more about temperaments see Llewelyn Lloyd's *Intervals, scales and temperaments* (London: Macdonald, 1963), J. Murray Barbour's *Tuning and temperament, a historical survey* (East Lansing: Michigan State College Press, 1953), and Owen Jorgensen's *Tuning* (East Lansing: Michigan State College Press, 1991). Also see Mark Lindley's articles in *The New Grove Dictionary of Music and Musicians*, ed. Stanley Sadie, (London: Macmillan, 1980) under "Interval" (9:277-9) and "Temperaments" (18:660-74) and his *Lutes, viols and temperments* (Cambridge, 1984).

7 With the ascendence of the third as the predominant interval in the 15th century, Pythagorean tuning was gradually replaced by meantone temperaments. Depending on the variety of meantone, the thirds were pure or nearly pure and the fifths were flat and the fourths were sharp (both only slightly). The flat fourths and sharp fifths were more than compensated for by the sweetness of the thirds. Meantone temperaments also have two sizes of semitone, however, it is the diatonic semitone which is larger rather than the reverse as in Pythagorean tuning.

8 The cents values for these temperaments are be determined by adding fifths to fifths. For instance, in equal temperament, E-B is 700c. To arrive at the cents value for the next fifth, F#, add another 700c. to the previous 700c. This yields 1400c. Finally, subtract 1200c. to keep the note within the octave and the result is an F# of 200c. For the next fifth, C#, add 700c. to the 200c. of the F# and the result is a C# of 900c., etc.

9 Since the harmonic at the 5th fret produces the pitch two octaves above the open string and octaves are pure in both equal temperament and Pythagorean tuning, in and of itself, the 5th fret harmonic does not present a problem.

10 Since the fourth is the inversion of the fifth, the equal-tempered fourth is higher by two cents, 500c. from 498c.

11 For an explanation of how the size of the major third is a function of the size of the fifth, see Appendix II.

12 For more on the syntonic comma, see Appendix II.

13 Hideo Kamimoto, *Complete Guitar Repair* (New York: Music Sales Corp., 1975), pp. 57-8.

14 As an experiment, you might try to tune your guitar's open strings to acoustically pure 4ths and an acoustically pure major 3rd. For more on this, see Appendix III.

15 This table is presented as if you were looking at your fingerboard from in front of the guitar. The numbers to the far left represent the strings while the numbers between the strings represent the size of the interval between each pair of strings. The figures in parentheses represent the interval in cents from that particular string to the 6th string.

16 One of the significant differences between equal temperament and many of the other historical temperaments, particularly the meantone temperaments, is that in those temperaments, there is usually a "wolf" fifth, i.e., a fifth that is so badly tuned as to be unusable. Rather than spreading the discrepancy (comma) around "equally," much, most, or all of it is deposited in one place. However, these temperaments are constructed so that the "wolf" fifth is placed in a remote key such as Ab. As a result, certain key areas sound exquisite (by virtue of pure or nearly pure thirds), while others are unusable.

17 "-" = the number of cents the string is narrow or flat and "+" = the number of cents the string is wide or sharp.

18 The assumption of tuning from the 6th string to the 1st string is for theoretical reasons only. For practical purposes it is better to tune from 1 to 6 for reasons that will be explained later.

19 The problem with Harmonic Method B is self-evident—the significant difference between the outer strings and two pure octaves. This needs no further elaboraton.

20 E and F major sound better than the open G chord because their lower octave is on the 4th string which is only 4c. flat, while the G chord has its octave on the 3rd string which is 6c. flat.

21 The perception of greater dissonance in the treble range is also magnified by the fact that our ears are naturally less sensitive to pitch differences in lower tessituras. For more on pitch discrimination, see John Backus, *The Acoustical Foundations of Music*, (New York: Norton, 1969), pp. 85-87, 110-111.

22 It is interesting to note that the very feature of Method A which makes it deceptive, i.e., its two perfect octaves between 6 and 1, is the same feature that makes it more useful for "ballpark tuning."

23 A common way to begin performances with other instrumentalists is to compare A's and then agree on what will be the "A *du jour*."

24 I am indebted to Clare Callahan for acquainting me with this tuning method.

25 And in the case of Method B, a pure third.

26 A quick and quiet I-IV-V-I works rather well, particularly when you are playing in an ensemble. I think it is important to hear more than just one chord before you start, since as we learned above, some chords might sound fine while others are unacceptable.

27 Since keyboard instruments are tuned less frequently than guitars, it is likely that they will be out of tune to some degree, however minute.

28 The formula used to derive the fretting factors is as follows:

$$\text{freting factor} = 1 - .5^{(x/1200)}$$

where x = the number of cents, e.g., for the 1st fret, 100; the 2nd fret, 200; etc. This formula is distinguished from formulas that are a function of fret number rather than cents by the fact that it can calculate the fret positions for any tuning or temperament as well as equal temperament. This formula is particularly useful for lutenists who are beginning to discover the advantages of setting their frets in the various meantone temperaments. See also John M. Linebarger, "Calculating Fret Positions for Non-standard Scale Lengths," *Soundboard*, Vol. XVIII/3, pp. 20-27 and several of the "Letters to the Editor" in Vol. XIX/2 of the same journal.

29 For more on a related subject (inharmonicity), see Backus, pp. 241-243.

30 Backus, p. 242. For a more detailed discussion of compensation see Kamimoto, pp. 60-1.

31 A compensated saddle is a one-piece saddle carved so that each string has its own mini-saddle. The effect is that each string is individually compensated for sharpening. Most electric guitars have individually adjustable bridges for each string.

32 It is also possible that the 12th fret is improperly positioned, but this is less likely because the 12 fret is the easiest fret to position.

33 The octaves are subtracted to keep the upper note within the octave.