Name: ____________________________ **Panther ID:** ______________

Exam 2 Calculus 3 Spring 2020

Important Rules:

A. Any electronic device (cell phone, calculator of any kind, smart-watch, etc.) should be turned off at the beginning of the exam and placed in your bag, NOT in your pocket. Electronic items, notes, texts, or formula sheets should NOT be used at any time during the examination. Concentrate on your own exam. Do not look at your neighbor's paper or try to communicate with your neighbor.

Violations of any type of this rule will lead to a score of zero on this exam, and possibly an automatic grade F for the course and a report for academic misconduct.

B. Unless otherwise mentioned, to receive full credit you must show your work. Answers which are not supported by work might receive no credit. Solutions should be concise and clearly written. Incomprehensible work might not be considered.

1. (12 pts) Consider the curve $\mathbf{r}(t) = 2\cos t\,\mathbf{i} + 3t\,\mathbf{j} - 2\sin t\,\mathbf{k}$, for $t \geq 0$.

(a) (6 pts) Sketch a graph of $\mathbf{r}(t)$ in 3d
and briefly describe the shape in words.

(b) (6 pts) Find the length of the curve when $0 \leq t \leq 2\pi$.

2. (12 pts) Circle True or False. No justification needed for these. (2 pts each - no partial credit)

(a) If $f_x(0,0)$ and $f_y(0,0)$ both exist, then f is continuous at $(0,0)$. **True** **False**

(b) If D is an open set in $\mathbf{R}^2$ then every point in D is an interior point. **True** **False**

(c) If f_x and f_y exist and are continuous on an open set, then $f(x,y)$ is differentiable on that set.

True **False**

(d) For a moving particle, the velocity vector and the acceleration vector are always perpendicular.

True **False**

(e) By definition, the binormal vector is $\mathbf{B}(t) = \mathbf{T}(t) \times \mathbf{N}(t)$. **True** **False**

(f) A circle with radius 5 has smaller curvature than one with radius 2. **True** **False**

3. (10 pts) Find the curvature $\kappa(t)$, for the plane curve given by $\mathbf{r}(t) = e^{3t}\ \mathbf{i} + e^{-t}\ \mathbf{j}$.

4. (16 pts) Let $f(x, y) = \sqrt{y - 2x - 1}$. Answer the following (4 pts each):

(a) Find the domain of f. Answer with words and/or formulas,
but also sketch the set representing the domain.

(b) Find the range of f.

(c) Find the boundary of the domain of f.

(d) Is the domain open or closed or neither ?

5. (10 pts) Find the equation of the tangent plane to the surface $2z - x^2 = 0$ at the point $(2, 0, 2)$.

6. (10 pts) Show that $f(x, y) = \ln\sqrt{x^2 + y^2}$ satisfies the Laplace equation $f_{xx} + f_{yy} = 0$.

7. (10 pts) Determine whether the limit exists. If so, find its value.

$$\lim_{(x,y)\to(0,0)} \frac{2xy}{3x^2 + 4y^2}$$

8. (16 pts) Consider $f(x, y, z) = \sin(xy) - yz + e^x z$.

(a) (8 pts) Find the direction (using a unit vector) in which the function increases most rapidly at the point $(1, 0, 1)$.

(b) (8 pts) If $x = ve^u$, $y = v\sin(u + v)$, and $z = v^2$, find f_u and f_v.

9. (12 pts) Choose ONE proof. If you do both, only the larger score will be considered for this problem, but the second proof may give some bonus towards a previous problem where your score is smaller.

(A) Find (with proof) the parametric equations for the ideal projectile motion.

(B) Prove that for a differentiable function $f(x, y)$, the gradient is normal to the level curves of f.