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Ex. 1

consider $f(x,y) = \begin{cases} \frac{xy}{x^2+y^2} & \text{if } (x,y) \neq (0,0) \\ 0 & \text{if } (x,y) = (0,0) \end{cases}$

Domain: all $\mathbb{R}^2$

a) Is $f(x,y)$ continuous at $(0,0)$?
b) Compute f_x and f_y at points $(x,y) \neq (0,0)$
c) Does $f_x(0,0)$ and $f_y(0,0)$ exist?
d) Is $f(x,y)$ differntable at $(0,0)$?

a) $\lim_{(x,y)\to(0,0)} f(x,y) \stackrel{?}{=} f(0,0) = 0$ $\quad \lim_{(x,y)\to(0,0)} \frac{xy}{x^2+y^2} = \lim_{r\to 0} \frac{r\cos\theta\, r\sin\theta}{r^2} = \sin\theta\cos\theta$ DNE

$f(x,y)$ is not continuous at $(0,0)$

b) $f_x = \frac{\partial f}{\partial x} = \frac{\partial}{\partial x}\left(\frac{xy}{x^2+y^2}\right) = \frac{y(x^2+y^2) - xy(2x)}{(x^2+y)^2} =$

$= \frac{x^2y + y^2 + 2x^2y}{(x^2+y^2)^2} = \frac{y^3 - x^2y}{(x^2+y^2)^2}$

$f_y = \frac{\partial}{\partial y}\left(\frac{xy}{x^2+y^2}\right) \underset{\uparrow}{=} \frac{-y^2x + x^3}{(x^2+y^2)^2}$

by the symmetry of x,y

c) $f_x(0,0) = \frac{\partial f}{\partial x}(0,0) \stackrel{\text{def}}{=} \lim_{h\to 0} \frac{f(h,0) - f(0,0)}{h} =$

$= \lim_{h\to 0} \frac{0 - 0}{h} = 0$

$f_y(0,0) = \lim_{h\to 0} \frac{f(0,h) - f(0,0)}{h} = 0$

d) No, because $f(x,y)$ is not continuous at $(0,0)$.

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d) $f(x,y) = xe^{\frac{y^2}{2}}$ Compute $\frac{\partial^5 f}{\partial x^2 \partial y^3} = 0$ (quickly)

The order of doing the partial der. does not matter

$f_x = e^{\frac{y^2}{2}}$
$f_{xx} = 0$

75 Find $\frac{\partial z}{\partial x}(1,1,1)$ if the eq. $xy + z^3x - 2yz = 0$ defines z as a function of (x,y)

$z = z(x,y)$ implicitely

$\frac{\partial}{\partial x}(xy + z^3x - 2yz) = \frac{\partial}{\partial x}(0)$

$y + 3z^2 \boxed{\frac{\partial z}{\partial x}} x + z^3 - 2y \boxed{\frac{\partial z}{\partial x}} = 0$

$\frac{\partial z}{\partial x}(3z^2x - 2y) = -y - z^3$

$\frac{\partial z}{\partial x} = -\frac{y + z^3}{3xz^2 - 2y} = \frac{y + z^3}{2y - 3xz^2}$

$\frac{\partial z}{\partial x}(1,1,1) = \frac{1+1}{2-3} = -2$

We say that $f(x,y)$ satisfies the laplace eq. if

$f_{xx} + f_{yy} = 0$

Ex. check that $f(x,y) = \ln\sqrt{x^2+y^2}$
$= \frac{1}{2}\ln(x^2+y^2)$

$f_x = \frac{1}{\cancel{2}}\frac{1}{x^2+y^2}\cancel{2}x = \frac{x}{x^2+y^2}$ $f_{xx} = \frac{1\cdot(x^2+y^2) - x(2x)}{(x^2+y^2)^2} = \frac{-x^2+y^2}{(x^2+y^2)^2}$

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$$f_{yy} = \frac{-y^2+x^2}{(x^2+y^2)^2} \qquad f_{xx}+f_{yy} = 0 \Rightarrow \text{harmonic Function}$$

$\boxed{14.4}$ ~ Chain rule ~

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Suppose $z = f(x,y)$

and assume $x = x(t)$ and $y = y(t)$

$z = z(t) = f(x(t), y(t))$

$$\boxed{\frac{dz}{dt} = \frac{\partial z}{\partial x} \cdot \frac{dx}{dt} + \frac{\partial z}{\partial y} \cdot \frac{dy}{dt}}$$

chain rule

At $t = t_0$ $\quad x(t_0) = x_0, \quad y(t_0) = y_0 \quad x = x(t) \quad y = y(t)$

$$\underset{z}{f(x,y)} \approx \underset{z_0}{f(x_0,y_0)} + f_x(x_0,y_0)\Delta x + f_y(x_0,y_0)\Delta y$$

$$\Delta z \approx \frac{\partial z}{\partial x}(x_0,y_0)\Delta x + \frac{\partial z}{\partial y}(x_0,y_0)\Delta y \quad / \div \Delta t$$

$$\frac{\Delta z}{\Delta t} \approx \frac{\partial z}{\partial x}\frac{\Delta x}{\Delta t} + \frac{\partial z}{\partial y}\frac{\Delta y}{\Delta t}$$

when $\Delta t \to 0$

$$\boxed{\frac{dz}{dt} = \frac{\partial z}{\partial x} \cdot \frac{dx}{dt} + \frac{\partial z}{\partial y}\frac{dy}{dt}}$$

Suppose $w = f(x,y,z)$ and $x = x(u,v)$, $y = y(u,v)$, $z = z(u,v)$

$w(u,v) = w = f(x(u,v), y(u,v), z(u,v))$

$$\frac{\partial z}{\partial u} = \frac{\partial w}{\partial x} \cdot \frac{\partial x}{\partial u} + \frac{\partial w}{\partial y} \cdot \frac{\partial y}{\partial u} + \frac{\partial w}{\partial z} \cdot \frac{\partial z}{\partial u}$$

$$\frac{\partial z}{\partial v} = \frac{\partial w}{\partial x} \cdot \frac{\partial x}{\partial v} + \frac{\partial w}{\partial y} \cdot \frac{\partial y}{\partial v} + \frac{\partial w}{\partial z} \cdot \frac{\partial z}{\partial v}$$

②/14.4 Let $w = x^2 + y^2$, suppose $x = \cos t + \sin t$,
$y = \cos t - \sin t$

Calculate: $\frac{dw}{dt}$ when $t = 0$

Solution 1

Use chain rule

$$\frac{dw}{dt} = \frac{\partial w}{\partial x} \cdot \frac{dx}{dt} + \frac{\partial z}{\partial y} \cdot \frac{dy}{dt}$$

$$\frac{dw}{dt} = 2x \cdot (-\sin t + \cos t) + 2y(-\sin t - \cos t)$$

$$= 2(\cos t + \sin t)(\cos t - \sin t) \quad 2(\cos t - \sin t)(\sin t + \sin$$

$$\frac{dw}{dt} = 2(\cos^2 t - \sin^2 t) - 2(\cos^2 t - \sin^2 t) = 0$$

$$\frac{dw}{dt} = 0 \Rightarrow w(t) \equiv \text{const.} \ \ast$$

Solution 2

$$w(t) = (\cos t + \sin t)^2 + (\cos t - \sin t)^2$$

$$= \cos^2 t + \cancel{2\cos t \sin t} + \sin^2 t + \cos^2 t - \cancel{2\sin t \cos t} + \sin$$

$$w(t) = 2 \Rightarrow \text{constant} \ \star$$

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$V = I \cdot R$ to

V I R
voltage, current, res

At the instant when $R = 600\ \Omega$

$I = 0.04$ A

$$\frac{dR}{dt} = 0.5\ \Omega/s \qquad \frac{dV}{dt} = 0.01\ V/s$$

Find $\frac{dI}{dt} = ?$

$$\frac{dV}{dt} = \frac{\partial V}{\partial I} \cdot \frac{dI}{dt} + \frac{\partial V}{\partial R} \cdot \frac{dR}{dt}$$

$$\boxed{\frac{dV}{dt} = R \cdot \frac{dI}{dt} + I \cdot \frac{dR}{dt}}$$