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Quiz 1 MAC-2313 Spring 2020

1. (3 pts) Match the following equations with the appropriate surface:

(i) $x^2 = 2y^2 + 3z^2$ (d)

(ii) $x^2 + 2y^2 - 3z^2 = 1$ (a)

(iii) $(x+1)^2 + 2(y-1)^2 + 3(z-2)^2 = 10$ (f)

(iv) $x = 1 + 2y^2 + 3z^2$ (e)

(v) $(x+1)^2 - 2(y-1)^2 - 3(z-2)^2 = 10.$ (c)

(vi) $2y^2 - 3z^2 = 1$ (b)

(a) hyperboloid with one sheet (b) hyperbolic cylinder (c) hyperboloid with two sheets (d) elliptic cone (e) elliptic paraboloid (f) ellipsoid

2. (8 pts) For both parts of this problem consider the lines L_1: $x = t$, $y = -t + 2$, $z = t + 1$, and L_2: $x = 2s + 2$, $y = s + 3$, $z = 5s + 6$.

(a) (4 pts) Show that L_1 and L_2 are intersecting and determine the point of intersection.

We should try to solve the system:

$$\begin{cases} t = 2s + 2 \\ -t + 2 = s + 3 \\ t + 1 = 5s + 6 \end{cases}$$

Adding the first two equations we get $2 = 3s + 5$ so $s = -1$. Substituting in the first, we get $t = 0$.

Observe that $\begin{cases} t = 0 \\ s = -1 \end{cases}$ satisfy all three equations (the 3rd should be checked) so the lines do intersect. Point of intersection is $P_0(0, 2, 1)$.

(plug in $t = 0$ in L_1, or $s = -1$ in L_2)

(b) (4 pts) Find the equation of the plane which contains both lines L_1 and L_2.

The directional vectors for L_1, L_2 respectively are

$$\vec{v}_1 = \langle 1, -1, 1 \rangle \text{ and } \vec{v}_2 = \langle 2, 1, 5 \rangle.$$

So a normal vector for our plane is $\vec{n} = \vec{v}_1 \times \vec{v}_2$

$$\vec{n} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & -1 & 1 \\ 2 & 1 & 5 \end{vmatrix} = -6\vec{i} - 3\vec{j} + 3\vec{k}$$

(of course, $-\frac{1}{3}\vec{v}_1 \times \vec{v}_2 = 2\vec{i} + \vec{j} - \vec{k}$ is also a normal vector)

As $P_0(0, 2, 1)$, from (a) is a point in the plane the equation of the plane is

$$-6(x-0) - 3(y-2) + 3(z-1) = 0 \quad | \div (-3)$$

or $2x + (y-2) + (z-1) = 0$ or $2x + y + z = 3$

any equivalent form is acceptable.