Name: Answer Key Panther ID: ____________

Quiz 02/20/2020 MAC-2313

1. (6 pts) Let $z = xy^2 + ye^{-x^2}$. Find each of the following:

(a) $\dfrac{\partial z}{\partial x} = y^2 + y\cdot e^{-x^2}\cdot(-2x) = y^2 - 2xye^{-x^2}$

(b) $\dfrac{\partial^2 z}{\partial x^2} = 0 - \left[2ye^{-x^2} + 2xye^{-x^2}\cdot(-2x)\right] = -2ye^{-x^2} + 4x^2ye^{-x^2} =$

$= 2ye^{-x^2}(2x^2-1)$

(c) $\dfrac{\partial^8 z}{\partial y^3 \partial x^5} = 0$ it is a mixed higher order partial, so we can choose the order to do the derivatives. It's much easier to differentiate w.r.t. y first

$\dfrac{\partial z}{\partial y} = 2xy + e^{-x^2}$ $\quad \dfrac{\partial^2 z}{\partial y^2} = 2x \Rightarrow \dfrac{\partial^3 z}{\partial y^3} = 0 \Rightarrow \dfrac{\partial^8 z}{\partial y^3 \partial x^5} = \dfrac{\partial^8 z}{\partial x^5 \partial y^3} = 0$

2. (5 pts) The temperature at a point (x, y) on a metal plate in the xy-plane is given by $T(x, y) = 2x^2 + 3x - 3y^2$ degrees Celsius.

(a) (1 pt) What is the temperature at the point $(2, 0)$?

$T(2,0) = 2\cdot 2^2 + 3\cdot 2 - 0 = 14\,^\circ C$

(b) (1 pt) An ant is moving on the metal plate so that at time t (in seconds) its position is given by $(x(t) = 2\cos t, y(t) = \sin t)$. In one sentence, describe the trajectory of the ant.

It is the ellipse $\frac{x^2}{4} + y^2 = 1$ followed counter-clock wise.

(c) (3 pts) What temperature does the ant experience at $t = \pi/2$ seconds and what is the rate of change of temperature with respect to time at that moment?

at $t = \frac{\pi}{2}$ s the position of the ant is $x(\frac{\pi}{2}) = 0$, $y(\frac{\pi}{2}) = 1$, so $T(0,1) = -3\,^\circ C$ is the temperature the ant feels at that moment.

$\dfrac{dT}{dt} \underset{\text{chain rule}}{=} \dfrac{\partial T}{\partial x}\cdot\dfrac{dx}{dt} + \dfrac{\partial T}{\partial y}\cdot\dfrac{dy}{dt} = (4x+3)(-2\sin t) - 6y\cdot\cos t =$

$= (8\cos t + 3)(-2\sin t) - 6\sin t\cdot\cos t$

so $\left.\dfrac{dT}{dt}\right|_{t=\frac{\pi}{2}} = (8\underbrace{\cos\tfrac{\pi}{2}}_{0} + 3)(-2\underbrace{\sin\tfrac{\pi}{2}}_{1}) - 6\underbrace{\sin\tfrac{\pi}{2}\cos\tfrac{\pi}{2}}_{0} = -6\,^\circ C/s$