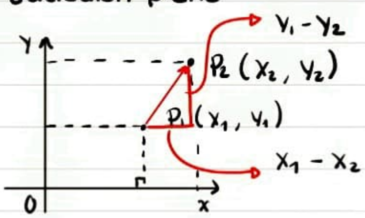
08/24/2021

Lecture 1

$\mathbb{R}^2 = \{(x, y) \mid z, y \in \mathbb{R}\}$

↳ Eucledian plane

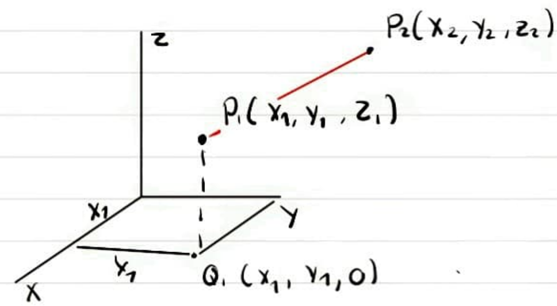


$$d(P_1, P_2) = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$z = f(x, y) = x^2 + y^2$

$\mathbb{R}^3 \Rightarrow \{(x, y, z) (x, y, z \in \mathbb{R})\}$

↳ Eucledian 3 space



$P_1(x_1, y_1, z_1)$ $P_2(x_2, y_2, z_2)$

$$d(P_1, P_2) = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

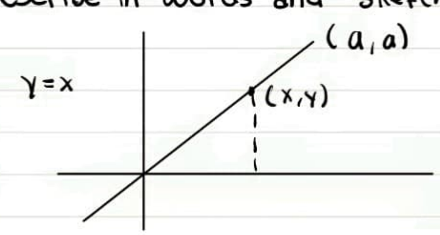
$\mathbb{R}^n = \{(a_1, a_2, \ldots a_n) \mid a_1, a_2, \ldots a_n \in \mathbb{R}\}$

↳ Euclidean space with n dimensions

$A(a_1, a_2, \ldots, a_n)$ $B(b_1, b_2, \ldots, b_n)$

$$d(A, B) = \sqrt{(b_1 - a_1)^2 + (b_2 - a_2)^2 + (b_n - a_n)^2}$$

Describe in words and sketch in $\mathbb{R}^2$ the sets corresponding to



$y = mx + b$

graph in $\mathbb{R}^2$ is a line

$z^2 + y^2 = 4$

↓

circle with center at $(0, 0)$ and radius 2.

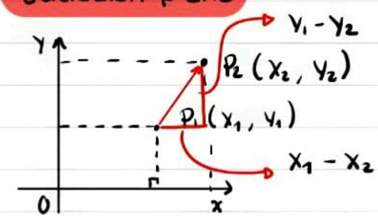
Give the equation for a circle with center $C(a, b)$ and radius 2. $(x - a)^2 + (y - b)^2 = r^2$

08/24/2021

Lecture 1

$\mathbb{R}^2 = \{(x, y) \mid z, y \in \mathbb{R}\}$

Eucledian plane

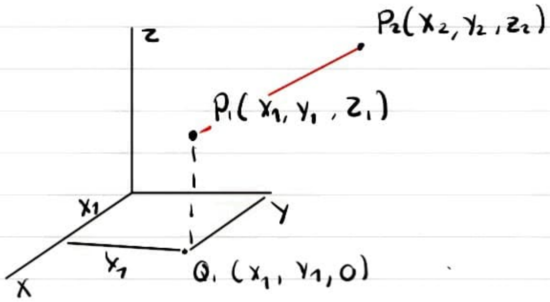


$$d(P_1, P_2) = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$z = f(x, y) = x^2 + y^2$

$\mathbb{R}^3 \Rightarrow \{(x, y, z) \mid x, y, z \in \mathbb{R}\}$

Eucledian 3 space



$P_1(x_1, y_1, z_1)$ $P_2(x_2, y_2, z_2)$

$$d(P_1, P_2) = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

$\mathbb{R}^n = \{(a_1, a_2, \dots a_n) \mid a_1, a_2, \dots a_n \in \mathbb{R}\}$

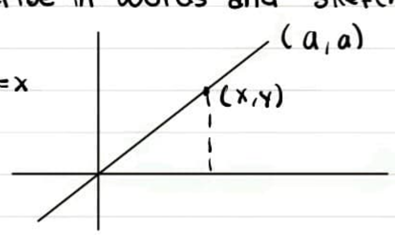
Euclidean space with n dimensions

$A(a_1, a_2, \dots, a_n)$ $B(b_1, b_2, \dots, b_n)$

$$d(A, B) = \sqrt{(b_1 - a_1)^2 + (b_2 - a_2)^2 + (b_n - a_n)^2}$$

Describe in words and sketch in $\mathbb{R}^2$ the sets corresponding to

$y = x$



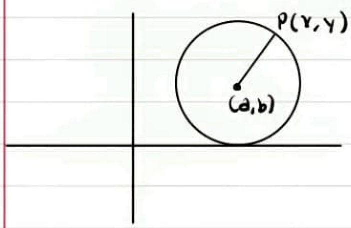
$y = mx + b$

graph in $\mathbb{R}^2$ is a line

$z^2 + y^2 = 4$

circle with center at $(0,0)$ and radius 2

Give the equation for a circle with center $C(a, b)$ and radius 2. $(x-a)^2 + (y-b)^2 = r^2$



$d(P,C) = \sqrt{(x-a)^2 + (y-b)^2} = r$

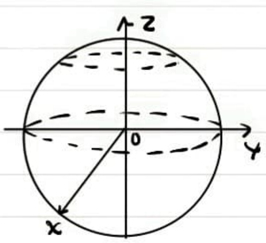
In $\mathbb{R}^3$

Equation of a sphere with center $C(a,b,c)$ and radius r

$P(x,y,z)$

$$\sqrt{(x-a)^2 + (y-b)^2 + (z-c)^2} = r^2$$

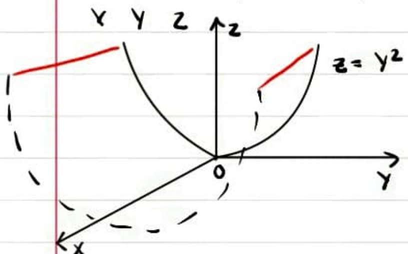
if $C = O(0,0,0)$, because $x^2 + y^2 + z^2 = r^2$



Describe in words and sketch in $\mathbb{R}^3$ the set of points (x,y,z) satisfy $z = y^2$ (*)

Example of points $P(x,y,z)$ satisfy (*)

$(0,2,4)$, $(5,2,4)$ $(a_1,2,4)$
x y z

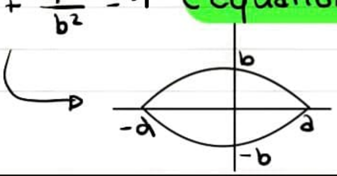


(*) a parabolic cylinder along the x-axes.

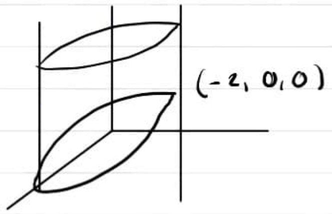
In $\mathbb{R}^3$ describe and sketch $P(x,y,z)$

$x^2 + 4y^2 = 4$ elliptical cylinder along the z-axis

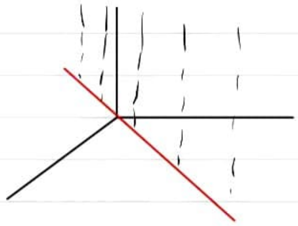
$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ (equation of an ellipse in the plane $\mathbb{R}^2$



$\frac{x^2}{4} + y^2 = 1 \Leftrightarrow \frac{x^2}{2^2} + \frac{y^2}{1^2} = 1$



Describe the set of points $P(x, y, z)$ in 3 spaces satisfy $y = x$



$y = x \rightarrow$ a plane containing the z axis and the line $y = z$ from zy-plane

Problem

Describe the set of points $P(x, y, z)$ in 3 space satisfying

$$x^2 + y^2 - 4y + z^2 = 5$$

$$(A \pm B)^2 = A^2 \pm 2AB + B^2$$

$$y^2 - 4y = y^2 - 2y \cdot 2$$

$$y^2 - 4y = y^2 - 2y \cdot 2 + 2^2 - 2^2$$

$$= (y-2)^2 - 4$$

$$x^2 + y^2 - 2y \cdot 2 + 4 - 4 + z^2 = 5$$

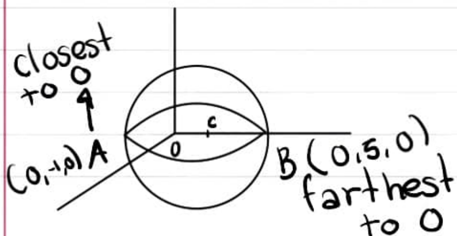
$$x^2 + (y-2)^2 + z^2 = 5 + 4$$

$x^2 + (y-2)^2 + z^2 = 9 \rightarrow$ Sphere with center at $(0, 2, 0)$ and radius 3

R^3

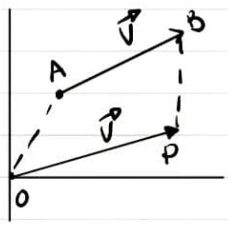
B) Suppose a bug is moving on the sphere of part (a)

How close to the origin can the bug get?



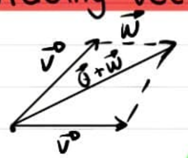
13.1, 13.2 Vectors in $\mathbb{R}^2$ and $\mathbb{R}^3$

Vectors → directed segments



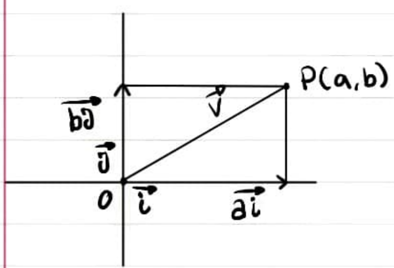
$\overrightarrow{AB}$ position vectors have their distant point at the origin

Adding vectors (geometrically)



↳ parallelogram rule

(Also the triangle rule for tail to tip rule)



$\vec{v} = \langle a, b \rangle = a\vec{i} + b\vec{j}$

$\vec{w} = \langle c, d \rangle = c\vec{i} + d\vec{j}$

$\vec{v} + \vec{w} = (a\vec{i} + b\vec{j}) + (c\vec{i} + d\vec{j}) = (a+c)\vec{i} + (b+d)\vec{j}$

Multiplying vectors by a scalar

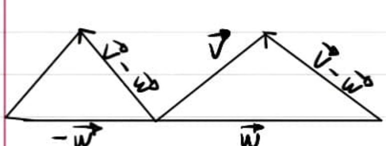
$\vec{v} \quad -3\vec{v} \leftarrow$

Subtracting vectors

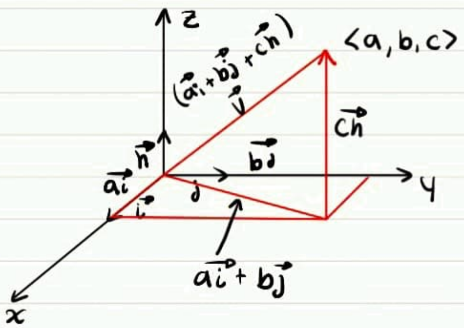
$\vec{v} = \langle a, b \rangle$
$\vec{w} = \langle c, d \rangle$
$\vec{v} - \vec{w} = \langle a-c, b-d \rangle$

Geometrically

$\vec{v} + (-\vec{w})$



Vectors in 3 spaces



$\vec{i} \langle 1,0,0 \rangle$

$\vec{j} \langle 0,1,0 \rangle$

$\vec{h} \langle 0,0,1 \rangle$

$\vec{v} = \langle a,b,c \rangle = a\vec{i} + b\vec{j} + c\vec{h}$