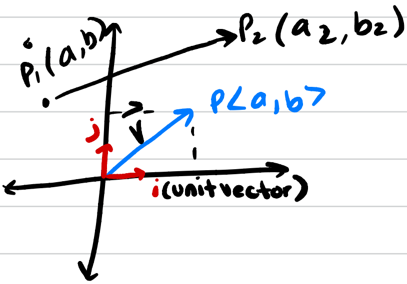
more on vectors: in $\mathbb{R}^2$ or $\mathbb{R}^3$ 8/26/21



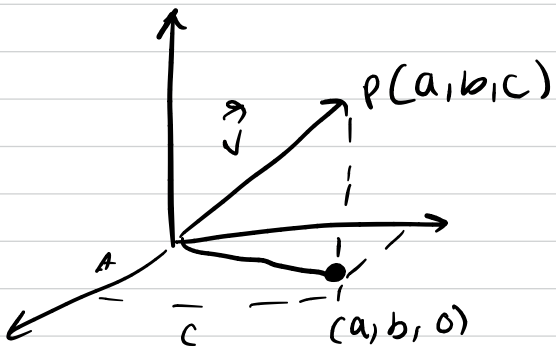
$\vec{v} = \langle a,b\rangle = a\hat{i} + b\hat{j}$

$P_1P_2 = \langle a_2 - a_1,\ b_2 - b_1\rangle$

Length of a Vector (Mag) North)

$\vec{v}$ in $\mathbb{R}^3$ $\vec{v} = \langle a,b,c\rangle$

$|\vec{v}| = \sqrt{a^2+b^2+3^2}$



$|\vec{v}| = \sqrt{a^2+b^2+c^2}$

- $\vec{v} \geq 0$ and $|\vec{v}| = 0$ if and only if the vector is the 0 vector

$\vec{v} = \vec{0}$

- $\vec{v}_1 = \langle a_1, b_1, c_1 \rangle$
- $\vec{v}_2 \langle a_2, b_2, c_2 \rangle$

$\vec{v}_1 + \vec{v}_2 = \langle a_1 + a_2, b_1 + b_2, c_1 + c_2 \rangle$

Geometically: its obtained by parallelogram & triangle Rule.

- $\alpha \cdot \vec{v}_1 = \alpha \cdot \langle a_1, b_1, c_1 \rangle =$

$\alpha a_1, \alpha b_1, \alpha c_1$

geometrically $\alpha \cdot \vec{v}$ is parallel to $\vec{v}_1$ and has length $|\alpha| \cdot |v_1|$ compared to $|v_1|$

if Negative it goes in the opposite direction

- Properties:

- $\vec{v}_1 + \vec{v}_2 = \vec{v}_2 + \vec{v}_1$
- $|\alpha \cdot \vec{v}| = |\alpha| |\vec{v}|$
- $(\vec{v}_1 + \vec{v}_2) + \vec{v}_3 = \vec{v}_1 + (v_2 + v_3>$
- $\vec{v} + \vec{0} = \vec{0} + \vec{v} = \vec{v}$
- $(-1) \cdot \vec{v} = -\vec{v}$
- $(\alpha \cdot \beta) \cdot \vec{v} = \alpha \cdot (\beta \cdot \vec{v})$

Intro to 13.3 Dot product

- If $\vec{v}_1 = \langle a_1, b_1, c_1 \rangle$ and $\vec{v}_2 = \langle a_2, b_2, c_2 \rangle$ are vectors in $\mathbb{R}^3$ then $\vec{v}_1 \cdot \vec{v}_2 \overset{\text{def}}{=} a_1 \cdot a_2 + b_1 \cdot b_2 + c_1 \cdot c_2$

it captures both length of vectors and angles between vectors

- If $\vec{v} = \langle a, b, c \rangle = a\vec{i} + b\vec{j} + c\vec{k}$

$$\vec{v} \cdot \vec{v} = a^2 + b^2 + c^2 = |\vec{v}|^2 \geq 0$$

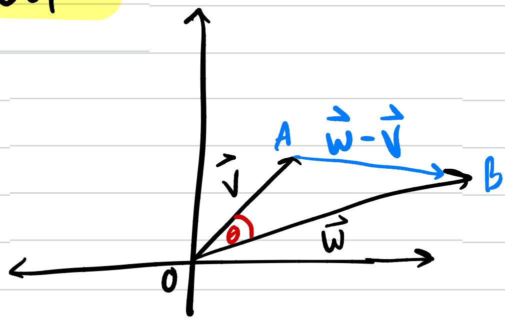
$$|\vec{v}| = \sqrt{\vec{v} \cdot \vec{v}}$$

- Theorm: for any vectors $\vec{v}$ and $\vec{w}$ in $\mathbb{R}^3$ (or $\mathbb{R}^2$)

$$\vec{v} \cdot \vec{w} = |\vec{v}| \cdot |\vec{w}| \cdot \cos\theta$$

* where θ is the angle betwen vectors

proof:



* $\vec{v}$ and $\vec{w}$ are orthogonal if and only if their dot product is equal to 0.

- Apply the law of Cos on ΔAOB, $|AB|^2 = |OA|^2 + |OB|^2 - 2|OA| \cdot |OB| \cos\theta$

Vector form:

$$|\vec{w} - \vec{v}|^2 = |\vec{v}|^2 + |\vec{w}|^2 - 2|\vec{v}| \cdot |\vec{w}| \cdot \cos\theta$$

$$(\vec{w} - \vec{v}) \cdot (\vec{w} - \vec{v})$$

$$\vec{w} \cdot \vec{w} - (\vec{w} \cdot \vec{v}) - (\vec{v} \cdot \vec{w}) + \vec{v} \cdot \vec{v}$$

properties of
• product
$\vec{v} \cdot \vec{w} = \vec{w} \cdot \vec{v}$

$$|\vec{w}|^2 - 2\vec{v} \cdot \vec{w} + |\vec{v}|^2$$

* Plug back into

$$\cancel{|\vec{w}|^2} - 2\vec{v} \cdot \vec{w} + \cancel{|\vec{v}|^2} = \cancel{|\vec{v}|^2} + \cancel{|\vec{w}|^2} - 2|\vec{v}| |\vec{w}| \cdot \cos\theta$$

$$-\cancel{2}\vec{v} \cdot \vec{w} = -\cancel{2}|\vec{v}| \cdot |\vec{w}| \cdot \cos\theta$$

$$\vec{v} \cdot \vec{w} = |\vec{v}| |\vec{w}| \cdot \cos\theta$$

• $\cos\theta = \dfrac{\vec{v} \cdot \vec{w}}{|\vec{v}| \cdot |\vec{w}|}$ if you want θ do arccos.

Exercise 1:

Given $\vec{v} = \langle 3,-1,2 \rangle$ and $\vec{w} = \langle 4,0,-3 \rangle$

find the following:

(a.) Unit vector with the same direction as $\vec{w}$

(b.) A vector $\vec{a}$ of length 6 of opposite direction as w.

(c.) The Angle between $\vec{v}$ and $\vec{w}$

(d.) A nonzero orthogonal Vector $\vec{b}$ to $\vec{v} + \vec{w}$

(a.) $\langle 3,-1,2 \rangle$ and $\vec{w} \langle 4,0,-3 \rangle$

$$|\vec{w}| = \sqrt{4^2+0^2+(-3)^2} = \sqrt{16+9} = 5$$

$$\frac{4}{5}, \frac{0}{5}, \frac{-3}{5} = \langle 4/5, 0, -3/5 \rangle \text{ or } \vec{u} = \frac{1}{|\vec{w}|} \cdot \vec{w}$$

$$\boxed{+\frac{1}{5}\vec{w}}$$

(b.) $\vec{a} = -6 \cdot \frac{1}{|\vec{w}|} \cdot \vec{w} = -6\vec{u} = -6 \langle \quad \rangle$

(c.) $\vec{v} \cdot \vec{w} = |\vec{v}| \cdot |\vec{w}| \cdot \cos\phi$

* dot product is also called scalar product

$\langle 3, -1, 2 \rangle \cdot \langle 4, 0, -3 \rangle$ $\quad \sqrt{3^2 + 1^2 + 2^2}$

$12 + 0 - 6 > = |5| |\sqrt{14}| \cdot \cos\phi$

$\cos^{-1}\left(\frac{6}{5 \cdot \sqrt{14}}\right) = \phi$

(d.) Need to find $\vec{b}$ so that $\vec{b} \cdot (\vec{v} + \vec{w}) = 0$

$\vec{v} + \vec{w} = \langle 3, -1, 2 \rangle + \langle 4, 0, -3 \rangle$

$\langle 7, -1, -1 \rangle$

$\vec{b} = \langle x, y, z \rangle$

↓ Dot product result

$7x - y - z = 0$

* has infinite solutions.

but x can be 0
y can be 5
z can be −5

$\vec{b} = \langle 0, 5, -5 \rangle$

↓

one possibility