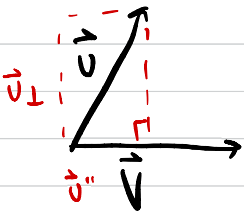
Quiz on Thursday 13.3 & 13.4 8/31/21

Still from 13.3

Orthogonal projections



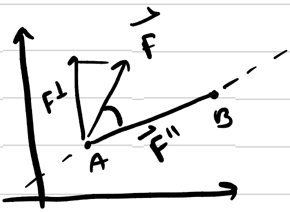
$\vec{U} = \vec{U}_{\parallel} + \vec{U}_\perp = \alpha \cdot \vec{V} + \vec{U}^\perp$ * take dot with $\vec{V}$

$\vec{U}_{\parallel} = \text{Proj}_{\vec{V}} \vec{U} = \alpha \cdot \vec{V}$

$\vec{U} \cdot \vec{V} = \alpha \vec{V} \cdot \vec{V}$

$\dfrac{\vec{U} \cdot \vec{V}}{\vec{V} \cdot \vec{V}} = \alpha \quad = \text{Proj}_{\vec{V}} = \dfrac{\vec{U} \cdot \vec{V}}{\vec{V} \cdot \vec{V}} \cdot \vec{V}$

Work of a constant force

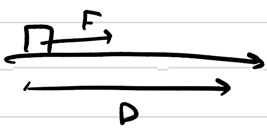


$\vec{F} = F_\perp + F_{\parallel}$

$W = |\vec{F}_{\parallel}| \cdot |AB| \cdot \cos\theta$ Also the dot product.

$|\vec{F}| \cdot |AB| \cdot \cos\theta = \text{work}$

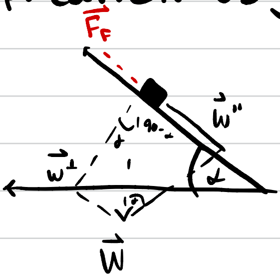
$W = \vec{F} \cdot \vec{AB}$ → (displacement vector)



if the constant force is in the direction of the motion the work is

$|F| \cdot |D| = \text{work}$

Application: Object on an incline plane



$\vec{F}_F$ = friction force

$$|\vec{F}_F| = \mu \cdot |\vec{w}_\perp|$$

μ ↑ Coefficient of friction

Object is in equilibrium if the two forces $|\vec{w}_{\parallel}| \leq |\vec{F}_F|$

$$|\vec{w}''| = |\vec{w}| \cdot \sin\alpha$$

$$|\vec{w}_\perp| = |\vec{w}| \cdot \cos\alpha$$

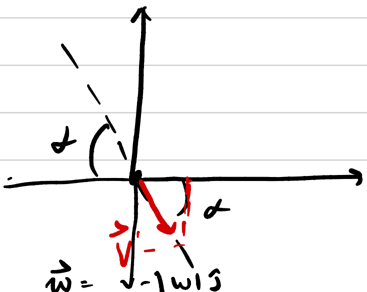
$$|F_F| = \mu \cdot |\vec{w}| \cdot \cos\alpha$$

$$\cancel{|\vec{w}|} \sin\alpha \leq \mu \cdot \cancel{|\vec{w}|} \cos\alpha$$

$$\frac{\sin\alpha}{\cos\alpha} \leq \mu$$

$\mu \geq \tan\alpha$ * Condition for the object to not move *

With orthogonal projection:



* $\vec{v}$ is a unit vector in the direction of the incline plane *

$$|\vec{v}| = 1$$

$$\vec{v} = (\cos\alpha)\hat{i} - \sin(\alpha)\hat{j}$$

$\vec{w} = \vec{w}_{\parallel} + \vec{w}_{\perp}$

$\vec{w}_{\parallel} = \text{proj}_{\vec{v}} \vec{w} = \dfrac{\vec{w}\cdot\vec{v}}{\vec{v}\cdot\vec{v}} \cdot \vec{v}$

as $|\vec{v}| = 1$ =

$\vec{w} = |\vec{w}| \cdot \hat{j}$

$\vec{v} = (\cos\alpha)\hat{i} - \sin(\alpha)\hat{j}$

$\vec{w}\cdot\vec{v} = |\vec{w}| \cdot \sin\alpha$

$\vec{w}_{\parallel} = \dfrac{|\vec{w}| \cdot \sin\alpha}{1} \cdot \vec{v}$

$|\vec{w}_{\parallel}| = |w| \sin\alpha$

$\vec{w}_{\perp} = \vec{w} - \vec{w}_{\parallel}$

13.4 Cross-product (vector product)

*makes sense only in R^3

Definition: $\vec{u} \times \vec{v} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \end{vmatrix} = \begin{vmatrix} u_2 & u_3 \\ v_2 & v_3 \end{vmatrix} \hat{i} - \begin{vmatrix} u_1 & u_3 \\ v_1 & v_3 \end{vmatrix} \hat{j} + \begin{vmatrix} u_1 & u_2 \\ v_1 & v_2 \end{vmatrix}$

$(u_2 v_3 - u_3 \cdot v_2)\hat{i} - (u_1 v_3 - u_3 v_1)\hat{j} + (u_1 v_2 - u_2 v_1)k$

$u = \langle 1, 2, 3 \rangle \quad v = \langle 4, 5, 6 \rangle$

$u = \hat{i} + 2\hat{j} + 3k \quad v = 4\hat{i} + 5\hat{j} + 6k$

$$u \times v = \begin{vmatrix} i & j & k \\ 1 & 2 & 3 \\ 4 & 5 & 6 \end{vmatrix} = \begin{vmatrix} 2 & 3 \\ 5 & 6 \end{vmatrix} \hat{i} - \begin{vmatrix} 1 & 3 \\ 4 & 6 \end{vmatrix} + \begin{vmatrix} 1 & 2 \\ 4 & 5 \end{vmatrix}$$

$$(12-15)\hat{i} - (6-12)\hat{j} + (5-8)\hat{k}$$

$$-3\hat{i} + 6\hat{j} - 3k$$

- $\vec{u} \cdot |u \times v| = -3 + 2(6) + 3(-3)$

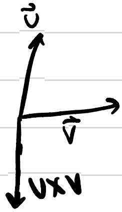
$-3 + 12 - 9 = 0$ $\quad \vec{u}$ is $\perp$ to $u \times v$

- $\vec{v} |u \times v| = 4(-3) + 5 \cdot 6 + 6 \cdot (-3) = 0$ $\quad \vec{v}$ is $\perp$ to $u \times v$

- Theorm: Geometric sig of cross product

A) if $\vec{u}$ and $\vec{v}$ are in R^3 $u \times v$ is a vector in R^3 $\perp$ to both $\vec{u}$ and $\vec{v}$, hence $\perp$ to the plane determined by $\vec{u}$ and $\vec{v}$.

B.)

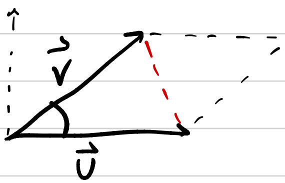


* the orientation of $u \times v$ will be decided by the right hand rule

c.) $|\vec{u}\times\vec{v}| = |\vec{u}|\cdot|\vec{v}|\cdot \sin\theta$ where θ is btwn u and v

$\downarrow$

Area of parallelogram determined by $\vec{u}$ and $\vec{v}$.



$$A_{Parallelogram} = 2\cdot A_{\Delta} = \frac{\cancel{2}\,\overbrace{|\vec{v}|\cdot\sin\theta}^{h}\,\cdot\overbrace{|\vec{u}|}^{b}}{\cancel{2}}$$

$$A_p = |\vec{v}|\,|\vec{u}|\cdot\sin\theta$$

$U\times V = -(V\times U)$ *opposite way*

$U\times(\vec{v}+\vec{w}) = (\vec{u}\times\vec{v}) + (\vec{u}\times\vec{w})$

$U\times(\alpha\vec{v}) = \alpha(\vec{u}+\vec{v})$

$\hat{i}\times\hat{j} = k = -\hat{j}\times\hat{i}$

$\hat{j}\times\hat{k} = \hat{i}$

$\hat{k}\times\hat{i} = \hat{j}$

*Triple Scalar product:

3 vectors in $\mathbb{R}^3$

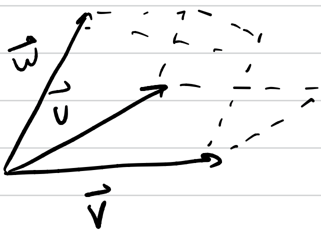
$\vec{u}$ $\vec{v}$ $\vec{w}$

$$\vec{w}\cdot(\vec{u}\times\vec{v}) = \begin{vmatrix} u_2 & u_3 \\ v_2 & v_3 \end{vmatrix} w_1 - \begin{vmatrix} u_1 & u_3 \\ v_1 & v_3 \end{vmatrix} w_2 + \begin{vmatrix} u_1 & u_2 \\ v_1 & v_2 \end{vmatrix} w_3$$

$$\begin{vmatrix} w_1 & w_2 & w_3 \\ u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \end{vmatrix} = \vec{w}\cdot(\vec{u}\times\vec{v})$$

triple scalar product

Geographically $|\vec{w}\cdot(\vec{u}\times\vec{v})|$ = volume of the 3-D box determined by $\vec{w}, \vec{u}, \vec{v}$

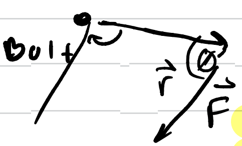


$$\vec{u}\cdot(\vec{u}\times\vec{v}) = 0$$

$$\vec{v}\cdot(\vec{u}\cdot\vec{v}) = 0$$

*if $\vec{u} \parallel \vec{v}$, then $\vec{u}\times\vec{v} = \vec{0}$

Cross product as Torque



$= \vec{T} = \vec{r} \times \vec{F}$

$|\vec{T}| = |\vec{r}\times\vec{F}| = |\vec{r}||\vec{F}|\cdot\sin\theta$

True or false questions

- 57 (13.4)

a.) The cross product of two nonzero vectors is always a nonzero vector. FALSE

example: $\vec{u} = 3\hat{i}$
$\vec{v} = 5\hat{i}$ $= \vec{0}$

b.) $|\vec{u} \times \vec{v}| \leq$ to both $|\vec{u}|$ and $|\vec{v}|$ FALSE

example: $u = 2\hat{i}$
$\vec{v} = 3\hat{j}$

$u \times v = 6k$

$|\vec{u}| = 2$ $6 > 2$ and $6 > 3$

$|\vec{v}| = 3$

$|\vec{u} \times \vec{v}| = 6$

c.) $|\vec{u} \times \vec{v}| \leq$ to $|\vec{u}| \cdot |\vec{v}|$