09/02/21

- Vectors are $\perp$ when the $\cdot$ product $= 0$
- Vectors are parallel when the $\times$ product $= 0$

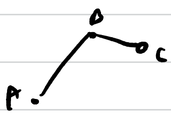
- Given two points $A(5,2,0)$, $B\langle 4,2,2\rangle$
  find vector v that is parallel to AB with length 10 and the same directions

$\vec{AB} = \langle -1, 0, 2\rangle$

$\vec{U}_{\text{unit vector parallel}} = \frac{1}{|AB|} \cdot AB$

$\vec{U} = 10 \cdot \frac{1}{|AB|} \cdot AB = \frac{10}{\sqrt{5}} \cdot \langle -1, 0, 2\rangle$

- Find the area of the triangle given by the points $A(5,2,0)$, $B(4,2,2)$ $C(1,2,3)$



$\vec{AB} = (-1, 0, 2)$

$\vec{AC} = (-4, 0, 3)$

$$AB \times AC = \begin{vmatrix} i & j & k \\ -1 & 0 & 2 \\ -4 & 0 & 3 \end{vmatrix} = \begin{vmatrix} 0 & 2 \\ 0 & 3 \end{vmatrix}\hat{i} - \begin{vmatrix} -1 & 2 \\ -4 & 3 \end{vmatrix}\hat{j} + \begin{vmatrix} -1 & 0 \\ -4 & 0 \end{vmatrix}\hat{k}$$

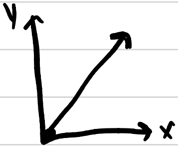
$$0\hat{i} - (-3 - {-8})\hat{j} + (0 - 0)\hat{j}$$

$$\frac{-5\hat{j}}{} = \sqrt{5^2} = \frac{5 \text{ Area of } \Delta}{2}$$

13.5 Lines and planes in 3 space

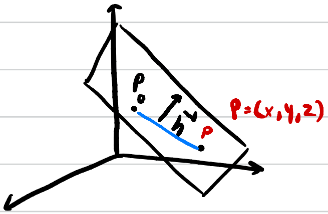
09/02/21

• Line in R^2 = $y = mx + b$



$ax + by = c$ → General Cartesian equation of all line in R^2

• Equation of a plane in R^3



Suppose we want to find the equation of a plane through a given point $P_0(x_0, y_0, z_0)$ and a given normal vector $\vec{n} = \langle a, b, c \rangle = a\hat{i} + b\hat{j} + c\hat{k}$

$$\vec{n} \perp P_0P = \vec{n} \cdot \overrightarrow{P_0P} = 0$$

$$\overrightarrow{P_0P} = (x - x_0,\ y - y_0,\ z - z_0)$$

$$a(x - x_0) + b(y - y_0) + c(z - z_0) = 0$$

Cartesian equation of a plane through $P_0(x_0, y_0, z_0)$ and of normal vector $\vec{n} = \langle a, b, c \rangle$

Ex: find the equation of the Plane that contains A(5,2,0) B(4,2,2) C(1,2,3)

find $AB \times BC$ which is $\vec{n}$

$$\downarrow$$

$$-5\hat{j} = \langle \underset{a}{0}, \underset{b}{-5}, \underset{c}{0} \rangle$$

take P_0 to be $A = (5, 2, 0)$
$\qquad\qquad\qquad\quad x_0 \;\; y_0 \;\; z_0$

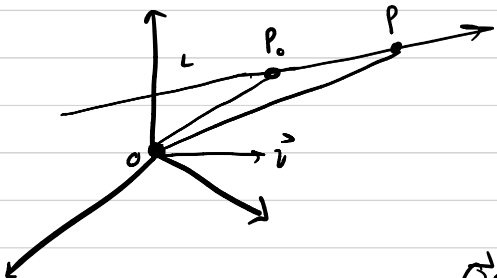
equation $= 0\cdot(x-5) - 5(y-2) + 0(z-0) = 0$

$$\frac{\cancel{-5}(y-2)}{\cancel{-5}} = \frac{0}{-5}$$

$$y-2=0$$

$$y=2$$

- equation of a line in 3-space
given a point $P_0(x_0, y_0, z_0)$ and has direction given by a vector $v = \langle a, b, c\rangle$



$\overrightarrow{P_0P} \parallel \vec{v} \iff \overrightarrow{P_0P}$ is a scalar multiple of v, t is a scalar

$\overrightarrow{P_0P} = t\cdot\vec{v}$

$$\vec{OP} - \vec{OP_0} = t\cdot\vec{v}$$

$$\vec{r} - \vec{r}_0 = t\cdot\vec{v}$$

$\vec{r} = t\cdot\vec{v} + r_0$ = vector equation of the line

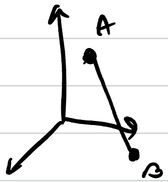
$\downarrow$

$$\langle x, y, z\rangle = \langle x_0, y_0, z_0\rangle + t\langle a, b, c\rangle$$

$x = x_0 + ta$
$y = y_0 + tb$
$z = z_0 + tc$

parametric equations of a line L that contains P_0 and has directional $\vec{v}$

ex: find the parametric equation of a line that goes through $A(2,3,1)$ $B(5,7,-1)$



$AB = \langle 3, 4, -2 \rangle$ Direction vector

$P_0 = A = (\overset{x_0}{2}, \overset{y_0}{3}, \overset{z_0}{1})$

$$\left.\begin{aligned} x &= 2 + 3t \\ y &= 3 + 4t \\ z &= 1 + -2t \end{aligned}\right\}$$ Parametric equation of the line

ex: Let $A(2,-3)$ $B(1,1)$
parametric equation of the line L through A & B

$\vec{V} = AB = \langle -1, 4 \rangle$ A is initial point $(\overset{x_0}{2}, \overset{y_0}{-3})$

$$\left.\begin{aligned} x &= x_0 + at = 2 - t \\ y &= y_0 + bt = -3 + 4t \end{aligned}\right\}$$ parametric equation of a line

eliminate Parameter

$t = 2 - x$

$y = -3 + 4(2 - x)$

$-3 + 8 - 4x = 5 - 4x$

$m = -4$

$(y - 1) = -4(x - 1)$

• two planes are parallel if and only if they are scalar multiples of one another

ex: Are $x+y+3z=1$ and $-2x-3y-6z=0$ parallel planes?

$n_1\ (1,1,3)$

$n_2\ (-2,-3,-6)$ — they are not scalar multiples ∴ they are not parallel planes.

(b.) find the equation of a plane through origin $(0,0,0)$ that is parallel to the plane $x+y+3z=1$

$n_1\ \langle 1,1,3\rangle$

$x+y+3z=0$

ex: Determine if the lines L_1: $x=1+2t$, $y=-3t$, $z=2+t$; L_2: $x=5-3s$, $y=1+s$, $z=2-s$ are parallel, intersecting, or skew.

$L_1\ \langle 2,-3,1\rangle \quad L_2\ \langle -3,1,-1\rangle$

$1+2t=5-3s$	$-3t=1+s$	$2+t=2-s$
$1+2\left(-\frac{1}{2}\right)=5-3\left(\frac{1}{2}\right)$	$-3(-s)=1+s$	$t=-s$
$1-1=5-\frac{3}{2}$	$3s=1+s$	$t=-(\frac{1}{2})$
$0\neq\frac{8}{2}$	$2s=1$	$t=-\frac{1}{2}$
	$s=\frac{1}{2}$	

∴ no intersection ∴ skew.