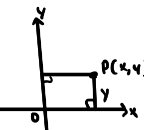
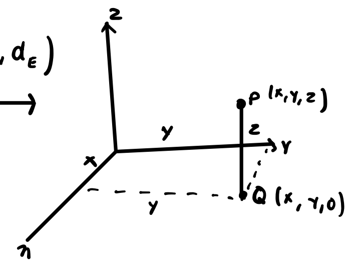
Calculus Multivariable

$z=f(x,y) \rightarrow$ a graph will be a surface in 3D



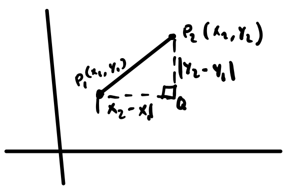
$\mathbb{R}^2$ Euclidean plane $\longrightarrow$ $(\mathbb{R})^2 = \{(x,y) \mid x, y \in \mathbb{R}\}, d_E)$

$\mathbb{R}^3$ Euclidean 3-space $(\mathbb{R})^3 = \{(x,y,z) \mid x,y,z \in \mathbb{R}\}, d_E)$ $\longrightarrow$



$\mathbb{R}^n$ Euclidean n-space $\mathbb{R}^n = \{(x_1, x_2, \dots, x_n) \mid x_i \in \mathbb{R}$ for each $\}_n$

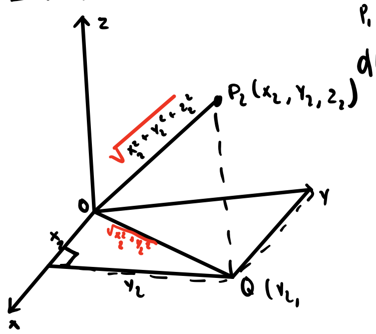
Euclidean distance in $\mathbb{R}^2$



$d(P_1,P_2) = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

$(P_1,P_2)^2 = (x_2-x_1)^2+(y_2-y_1)^2$

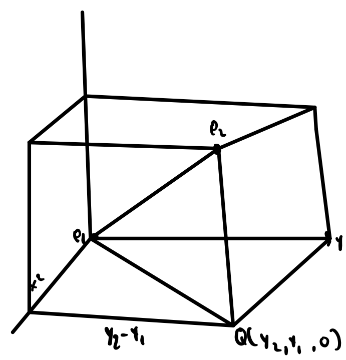
In $\mathbb{R}^3$



$P_1(x_1,y_1,z_1)$ $P_2(x_2,y_2,z_2)$

$d(P_1,P_2) = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}$

$d(0,P_2) = \sqrt{x_2^2+y_2^2+z_2^2}$



$\mathbb{R}^n = d(P,Q) = \sqrt{(y_1-x_1)^2+(y_2-x_2)^2+\dots+(y_n-x_n)^2}$

Question 1: Can you describe (in words) and draw the set of points (x,y,z) in 3-space that satifies:

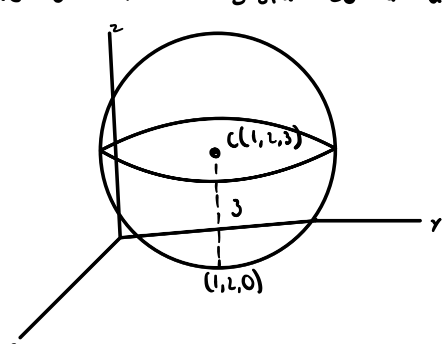
a) $(x^2-1)^2+(y-2)^2+(z-3)^2=9$

b) $y^2+z^2=4$

a) will be a sphere with center 1-3 and radius 3

If P is and arbitary point and $C,(1,2,3)$ $d(P,C) = \sqrt{(x^2-1)^2+(y-2)^2+(z-3)^2}$

Set of points $P(1,2,3)$ in 3 space so that $d(P,C)=3$



radius is the same as z coord center at 3

the y,z plane will intersect

xz plane also intersect

xy intersect in 1 point aswell

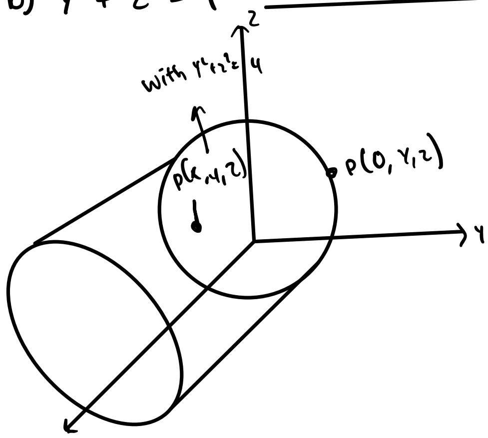
Ex Find the equation of the circle that corresponds to the intersection of the sphere $(x-1)^2+(y-2)^2+(z-3)^2=9$ with the yz plane

- The x coord is 0
- Solve the system

Circle with $r=\sqrt{8}$ center 0,2,3

$$\begin{bmatrix} (x-1)^2+(y-2)^2+(z-3)^2=9 \\ x=0 \end{bmatrix} \Rightarrow (0-1)^2+(y-2)^2+(z-3)^2=9 \Rightarrow (y-2)^2+(z-3)^2=8$$

b) $y^2+z^2=4$ → in 3 space this will be a cylinder along the x-axis

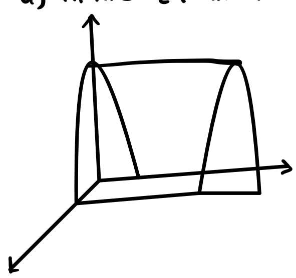


Ex 2: Describe and draw in 3D

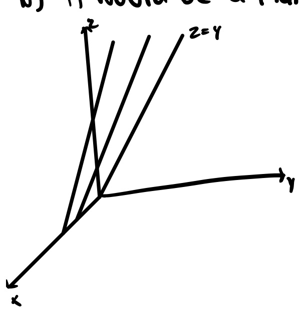
a) $z=4-x^2$

b) $z=y$

a) in the xz plane its a parabola in 3D its a parabolic cylinder



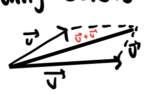
b) it would be a plane (a wall)



Vectors in 2D & 3D

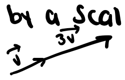
directed segments invariant under translation

Adding vectors → by Parallelogram rule or head to tail



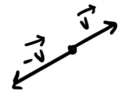
Multiplying vector by a scalar

$\vec{V}$ vector $3.\vec{V}$

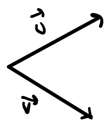


$\vec{av} = a \cdot \vec{v}$ same direction but magnitude changed by factor a

$-\vec{v}$



Subtract vectors



$\vec{v} - \vec{u} =$