Equation of a sphere in 3-D space with center $C(a,b,c)$ and radius r

$(x-a)^2+(y-b)^2+(z-c)^2=r^2$

$x^2+y^2+z^2-6x+4z=10$

$\Rightarrow x^2-2\cdot 3x+3^2-3^2+y^2+z^2+2\cdot 2z+2^2-2^2=10$

$\Rightarrow (x-3)^2+y^2+(z+2)^2-9-4=10$

$\Rightarrow (x-3)^2+y^2+(z-(-2))^2=23$

Sphere of center $(3,0,-2)$ and radius $\sqrt{23}$

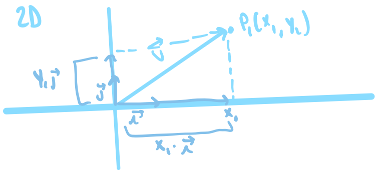
* if the radius is 0 its just a dot
* if too negative the sphere DNE

$x^2+y^2+z^2+ax+by+cz+d=0$

either a sphere, a dot, or DNE

Vectors in 2D and 3D Algebraically

2D



$\vec{i},\vec{j}$ are unit vectors along the x-axis, y-axis respectively

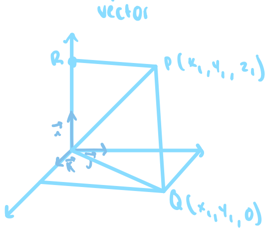
$\vec{v}=x_1\cdot\vec{i}+y_1\vec{j}=\langle x_1,y_1\rangle$

Vector that starts at origin is a position vector

$P_1 \longleftrightarrow \overrightarrow{OP_1}$

$(x_1,y_1)\quad \langle x_1,y_1\rangle = x_1\vec{i}+y_1\vec{j}$

vector



$\overrightarrow{OP_1}=\langle x_1,y_1,z_1\rangle = x_1\vec{i}+y_1\vec{j}+z_1\vec{k}$

$\overrightarrow{O_Q}=x_1\vec{i}+y_1\vec{j}$

$\overrightarrow{OP}=\overrightarrow{O_Q}+\overrightarrow{O_R}=x_1\vec{i}+y_1\vec{j}+z_1\vec{k}$

$P_1(x_1,y_1,z_1)\longleftrightarrow \overrightarrow{OP_1}=\langle x_1,y_1,z_1\rangle = x_1\vec{i}+y_1\vec{j}+z_1\vec{k}$

What does the following represent in 3D space?

$x^2+z^2-3z\le 0$

$\Downarrow$

complete the square

$x^2+z^2-2\,\frac{3}{2}z+(\frac{3}{2})^2-(\frac{3}{2})^2\le 0$

$\Downarrow$

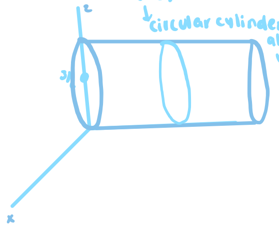
$x^2+(z-\frac{3}{2})^2-\frac{9}{4}\le 0$

$\Downarrow$

$x^2+(z-\frac{3}{2})^2\le \frac{9}{4}\rightarrow$ solid circular cylinder inside of along y-axis

if $x^2+(z-\frac{3}{2})^2=(\frac{3}{2})^2$

circular cylinder along y-axis



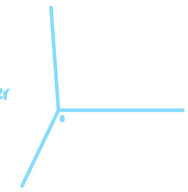
$x^2+z^2<0$

$\Downarrow$

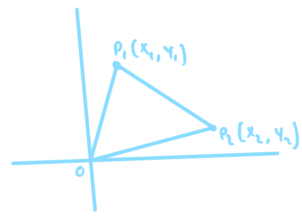
implies $x=0$ $z=0$

$\Downarrow$

the points on the y-axis



Vector with origin $\neq 0$



$\overrightarrow{P_1P_2}$

$\overrightarrow{OP_1}+\overrightarrow{P_1P_2}=\overrightarrow{OP_2}$

$\overrightarrow{P_1P_2}=\overrightarrow{OP_2}-\overrightarrow{OP_1}=\langle x_2-y_2\rangle-\langle x_1-y_1\rangle=\langle x_2-x_1,y_2-y_1\rangle$

$x_2\vec{i}+y_2\vec{j}-(x_1\vec{i}+y_1\vec{j})$

$\Downarrow$

$x_2\vec{i}+y_2\vec{j}-x_1\vec{i}-y_1\vec{j}=(x_2-x_1)\vec{i}-(y_2-y_1)\vec{j}$

in 3-space

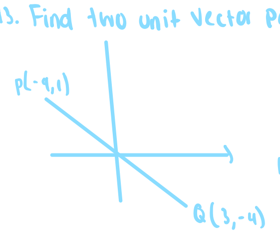
$\overrightarrow{Q_1Q_2}=\langle x_2-x_1,y_2-y_1,z_2-z_1\rangle$

$\overrightarrow{Q_1Q_2}=\overrightarrow{0Q_2}-\overrightarrow{0Q_1}$

Example 41-46

$P(-4,1)$ $Q(3,-4)$ $R(2,6)$

43. Find two unit vector parallel to $\overrightarrow{PQ}$



$\overrightarrow{PQ}=\langle 3-(-4),-4-1\rangle=\langle 7,-5\rangle$

$|\overrightarrow{PQ}|=|PQ|=\sqrt{(7)^2+(-5)^2}=\sqrt{74}$

length distance

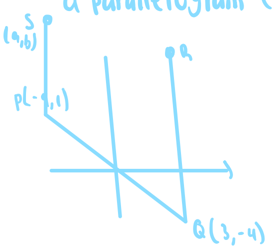
$\vec{U}=\frac{\overrightarrow{PQ}}{|\overrightarrow{PQ}|}=\frac{1}{|\overrightarrow{PQ}|}\cdot\overrightarrow{PQ}$

$1=\frac{1}{\sqrt{74}}\cdot\langle 7,-5\rangle=\langle \frac{7}{\sqrt{74}},\frac{-5}{\sqrt{74}}\rangle$

$\pm\langle \frac{7}{\sqrt{74}},\frac{-5}{\sqrt{74}}\rangle$

$-\vec{U}$ is also a unit vector parallel to $\overrightarrow{PQ}$

Ex2: Given P, Q, R as before, find a point so that P, Q, R, S is a parallelogram (in this order)



$\overrightarrow{QR}$ is parallel and of equal length with $\overrightarrow{PS}$

$\overrightarrow{QR} = \overrightarrow{PS} \implies \langle a-(-4), b-1\rangle$

$\parallel$ $\langle a+4, b-1\rangle$

$\langle 2-3, 6-(-4)\rangle$ $\Downarrow$ $S=(-5, 11)$

$\langle -1, 10\rangle$

$a+4=-1$ $a=-5$

$b-1=10$ $b=11$

Properties

$\vec{0} = \langle 0,0\rangle$

$\vec{v}+\vec{w} = \vec{w}+\vec{v}$

$\vec{v}+\vec{0} = \vec{v}$

$(\alpha\vec{v})\beta = (\beta\alpha)\vec{v}$

$0\cdot\vec{v} = \vec{0}$

$|\alpha\cdot\vec{v}| = |\alpha|\cdot|\vec{v}|$

abs value — length

Dot product (or scalar product)

2D: $\vec{v_1} = \langle a_1, b_1\rangle$

$\vec{v_2} = \langle a_2, b_2\rangle$

$\vec{v_1}\cdot\vec{v_2} = a_1a_2 + b_1b_2$

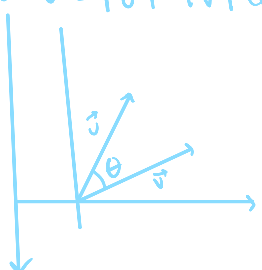
$\vec{v_1}\cdot\vec{v_1} = a_1^2 + b_1^2 = |\vec{v_1}|^2$ $\quad |\vec{v}| = \sqrt{\vec{v}\cdot\vec{v}}$

length of vector squared

3D: $a_1a_2 + b_1b_2 + c_1c_2$

Theorm #1: For any vectors $\vec{u}, \vec{v}$ in 2D or 3D

$\vec{u}\cdot\vec{v} = |\vec{u}|\cdot|\vec{v}|\cos\theta$ where θ is the angle between $\vec{u}$ and $\vec{v}$



Proof: cosine law

$\cos\theta = \dfrac{\vec{u}\cdot\vec{v}}{|\vec{u}|\cdot|\vec{v}|}$ $\longrightarrow$ Corollary $\vec{u} \perp \vec{v}, \iff \vec{u}\cdot\vec{v} = 0$