Name: Solution Key **Panther ID:** ______________________

Quiz 1 MAC-2313 Spring 2022

1. Consider the vectors $\mathbf{u} = \mathbf{i} - \mathbf{j}$ and $\mathbf{v} = \mathbf{i} - 2\mathbf{j} + 2\mathbf{k}$ in $\mathbf{R}^3$.

(a) (2 pts) Compute the dot product $\mathbf{u} \cdot \mathbf{v}$.

$$\vec{u} \cdot \vec{v} = 1 \cdot 1 + (-1) \cdot (-2) + 0 \cdot 2 = \boxed{3}$$

(b) (3 pts) Find the angle θ between $\mathbf{u}$ and $\mathbf{v}$.

$$\vec{u} \cdot \vec{v} = |\vec{u}|\,|\vec{v}| \cos\theta \quad \text{so} \quad \cos\theta = \frac{\vec{u} \cdot \vec{v}}{|\vec{u}| \cdot |\vec{v}|}$$

$$\cos\theta = \frac{3}{\sqrt{2} \cdot \sqrt{1^2 + 2^2 + 2^2}} = \frac{\cancel{3}}{\sqrt{2} \cdot \cancel{3}} = \frac{1}{\sqrt{2}} \Rightarrow \theta = \arccos\left(\frac{1}{\sqrt{2}}\right) = \boxed{\frac{\pi}{4} \text{ (or } 45^\circ)}$$

(c) (3 pts) Compute the cross product $\mathbf{u} \times \mathbf{v}$.

$$\vec{u} \times \vec{v} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & -1 & 0 \\ 1 & -2 & 2 \end{vmatrix} = \vec{i} \begin{vmatrix} -1 & 0 \\ -2 & 2 \end{vmatrix} - \vec{j} \begin{vmatrix} 1 & 0 \\ 1 & 2 \end{vmatrix} + \vec{k} \begin{vmatrix} 1 & -1 \\ 1 & -2 \end{vmatrix}$$

$$\text{so} \quad \vec{u} \times \vec{v} = -2\vec{i} - 2\vec{j} - \vec{k}$$

(d) (2 pts) Find the area of the parallelogram generated by $\mathbf{u}$ and $\mathbf{v}$.

$$= |\vec{u} \times \vec{v}| = \sqrt{(-2)^2 + (-2)^2 + (-1)^2} = \boxed{3}$$

(e) (2 pts bonus) Find the cartesian equation of the plane that contains the point $A(1, 2, 3)$ and is parallel to both vectors $\mathbf{u}$ and $\mathbf{v}$.

As $\vec{u}$ & $\vec{v}$ are parallel to the plane

$\vec{n} = \vec{u} \times \vec{v} = \langle -2, -2, -1 \rangle$ is a normal vector to the plane

Thus $(-2)(x-1) + (-2)(y-2) + (-1)(z-3) = 0$ is the equation of the plane

or $2(x-1) + 2(y-2) + (z-3) = 0$

or $2x + 2y + z = 9$