Worksheet 1-27 MAC 2313 – Spring '22 ___ **Names:** ______________________________

1. Match the following equations with the appropriate surface. Also, draw a sketch for the surface corresponding to equation (i).

(i) $x^2 = 2y^2 + 3z^2$

(ii) $x^2 + 2y^2 - 3z^2 = 1$

(iii) $(x+1)^2 + 2(y-1)^2 + 3(z-2)^2 = 10$

(iv) $x = 1 + 2y^2 + 3z^2$

(v) $(x+1)^2 - 2(y-1)^2 - 3(z-2)^2 = 10.$

(vi) $2y^2 - 3z^2 = 1$

(a) hyperboloid with one sheet (b) hyperbolic cylinder (c) hyperboloid with two sheets

(d) elliptic cone (e) elliptic paraboloid (f) ellipsoid

2. (8 pts) For both parts of this problem consider the lines L_1: $x = t$, $y = -t + 2$, $z = t + 1$, and L_2: $x = 2s + 2$, $y = s + 3$, $z = 5s + 6$.

(a) (4 pts) Show that L_1 and L_2 are intersecting and determine the point of intersection.

(b) (4 pts) Find the equation of the plane which contains both lines L_1 and L_2.

3. The position vector of a certain particle at time t is given by $\mathbf{r}(t) =< t\cos t \,,\, t\sin t \,,\, t >$.

(a) Show that the curve described by the particle lies on the cone $x^2 + y^2 = z^2$. The curve is a conical helix. Draw a sketch for this curve.

(b) Find the velocity vector $\mathbf{v}(t)$ and acceleration vector $\mathbf{a}(t)$.

(c) Find the speed $|\mathbf{v}(t)|$, and simplify the expression as much as possible.

(d) Set up the integral, but do not evaluate, to find the arclength of the curve in part (a) for $0 \leq t \leq 4\pi$.

4. Assume that $\mathbf{v}(t) =< \frac{1}{t+2} \,,\, 2t - 1 >$ is the velocity vector at time t of a certain particle.

(a) Find $\mathbf{v}'(2)$, and mention its physical significance.

(b) Find $\int_2^4 \mathbf{v}(t)\, dt$, and mention its physical significance.