Worksheet 02/24 - MAC 2313 **Group nr.** ____ **NAMES:** ____________________

1. Consider the function $f(x, y) = \ln(1 + 2x + 3y)$.

(a) Find the linearization $L(x, y)$ (or the local linear approximation) of $f(x, y)$ at $(0, 0)$.

(b) What is the equation of the tangent plane to the graph of $f(x, y)$ at $(0, 0)$?

(c) Use differentials, or the linearization you found in part (a), to estimate without calculator $f(1.02, 0.99)$.

2. According to the ideal gas law, the pressure, temperature, and volume of a confined gas are related by $P = k\frac{T}{V}$, where k is a constant. Use differentials to approximate the percentage change in pressure if the temperature of a gas is increased 3% and the volume is increased 5%.

3. Use chain rule to find the derivative $df/dt|_{t=0}$, for the function $f(x,y,z) = xy^2z^3$ along the path $< \ x(t) = e^t \ \cos t \ , \ y(t) = e^t \ \sin t \ , \ z(t) = t \ >$ at $t = 0$.

4. The temperature at a point (x,y) on a metal plate in the xy-plane is given by $T(x,y) = x^2 - \frac{3}{2}y^2 + 20$ degrees Celsius. Assume x, y are measured in centimeters.

(a) Suppose a bug is positioned initially at the point $(1,1)$ on the plate. What is the temperature at $(1,1)$?

(b) In which direction should the bug go (from $(1,1)$) to experience the most rapid *decrease* in temperature? Give your answer as a unit vector, but also as an (approximate) geographical direction. (Assume that the positive x-axis points East and that the positive y-axis points North.)

(c) Suppose next that on the same metal plate there is a heat-seeking ant, which always move in the direction corresponding to greatest increase in temperature. If the ant is initially at the origin, find the trajectory of this ant.