Worksheet 03/24/2022 MAC 2313 Group nr. ___ **Names:** ______________________

1. Circle if each of the following statements is true or false and then give a brief justification of your answer.

(a) The area of a region R in the xy-plane is given by $\displaystyle\int_R\int xy\,dA$ **True** **False**

Justification:

(b) For any continuous functions $f(x), g(x)$ on $[a, b]$,

$\displaystyle\int_a^b f(x)\cdot g(x)\,dx = \Big(\int_a^b f(x)\,dx\Big)\cdot\Big(\int_a^b g(x)\,dx\Big)$ **True** **False**

Justification:

(c) If $f(x)$ is continuous on $[a, b]$, $g(y)$ continuous on $[c, d]$ and R is the rectangle $R = [a, b] \times [c, d]$, then

$\displaystyle\int_R\int f(x)\cdot g(y)\,dA = \Big(\int_a^b f(x)\,dx\Big)\cdot\Big(\int_c^d g(y)\,dy\Big)$ **True** **False**

Justification:

(d) For a continuous function $f(x, y)$, $\displaystyle\int_0^1\int_{x^2}^x f(x, y)\,dy\,dx = \int_{x^2}^x\int_0^1 f(x, y)\,dx\,dy$ **True** **False**

Justification:

2. Set up an iterated double integral to represent each of the following:

(a) The area of the region R, where R is the triangle bounded by $y = 3$, $y = -x + 1$ and $y = x + 1$.

(b) The mass of a the thin plate covering the region R from part (a) and having density $\delta(x, y) = |xy|$.

3. (a) Evaluate the integral $\displaystyle\int_R\int \sin(y^3)\,dA$, where R is the region bounded by $y=\sqrt{x}$, $y=2$, and $x=0$.
Hint: Choose the order of integration carefully.

(b) Evaluate the integral by first reversing the order of integration: $\displaystyle\int_0^1\int_{4x}^4 e^{-y^2}\,dy\,dx$

4. Use polar coordinates to find the volume of the solid bounded inside the cylinder $x^2+y^2=9$ cut by the planes $z=0$ and $z=3-x$.

5. Evaluate $\displaystyle\int_R\int \frac{1}{1+x^2+y^2}\,dA$,

where R is the sector in the first quadrant bounded by $y=0$, $y=x$, and $x^2+y^2=4$.

6. (a) Use polar coordinates to find $\displaystyle\int_{\mathcal{R}}\int e^{-x^2-y^2}\,dA$ where $\mathcal{R}$ is the whole first quadrant of the xy-plane.

(b) Use part (a) to find the exact value of the improper integral $\displaystyle\int_0^{+\infty} e^{-x^2}\,dx$. This is an important integral in probability and statistics.