Worksheet 3/31 MAC 2313 – Spring '22 **Names:** ____________________

1. Use cylindrical coordinates (r, θ, z), to find the volume of the solid enclosed between the ellipsoid $4x^2 + 4y^2 + z^2 = 5$ and the elliptical (actually circular) paraboloid $z = x^2 + y^2$.

2. Find the mass of a spherical ball of radius a if the density is inversely proportional to the square of the distance from the center. That is, the density function is given by $\delta = \frac{k}{\rho^2}$, where k is a constant of proportionality. Use spherical coordinates.

3. (a) Use a triple integral in spherical coordinates to find the volume of the spherical cap enclosed inside the sphere $x^2 + y^2 + z^2 = 4$ and above the plane $z = 1$.

(b) This time compute the volume of the spherical cap in part (a) using a triple integral in cylindrical coordinates.

(c) Find the coordinates $(\bar{x}, \bar{y}, \bar{z})$ of the centroid of the spherical cap from part (a). Assume the density to be constant. *Hint:* Using symmetry, two of the coordinates are very easy to get. Just for the third, you'll need to compute an integral (and use the volume that you already know). For this integral, it is your choice now whether to use spherical or cylindrical coordinates.

4. An airplane flies the route Miami-New Orleans. Your task is to approximate the **shortest** travel distance (which determines the necessary fuel, etc.). Use that the radius of Earth is $R = 6370$ km (and assume that the plane flies close to the surface of the Earth).

(a) First solve the problem assuming that Miami and New Orleans have the same latitude; so assume the geographical coordinates of the two cites are Miami (30° N, 80° W) and New Orleans (30° N, 90° W).

(b) Now solve the problem using the more precise geographical coordinates Miami (25.74° N, 80.19° W) and New Orleans (29.95° N, 90.07° W).

(c) More generally, if P_1, P_2 are two points on the same sphere of radius R and the spherical coordinates of the two points are $P_1(R, \theta_1, \phi_1)$, $P_2(R, \theta_2, \phi_2)$ describe a way to find the shortest distance on the sphere between P_1 and P_2.

Hint 1: On a sphere, the geodesics (paths of shortest distance) are arcs on great circles, that is circles obtained from the intersection of the sphere with planes through the center of the sphere.

Hint 2: Suppose the sphere has center at the origin O and two points P_1, P_2 are given on the sphere. The geodesic between P_1 and P_2 is the (smaller) arc obtained from cutting the sphere with the plane through O, P_1, P_2.

Hint 3: The angle $\angle(P_1 O P_2)$ is important. Use the vectors $\mathbf{OP_1}$ and $\mathbf{OP_2}$ to find it.