Worksheet 4/05 - MAC 2313 **Group nr.** ____ **NAMES:** ______________________

1. Use the transformation $u = y/x$, $v = xy$, to find $\displaystyle\int_R\int xy^3\, dA$ over the region R in the first quadrant enclosed by $y = x$, $y = 3x$, $xy = 1$, $xy = 4$.

2. Using an appropriate change of variables, find $\displaystyle\int_R\int \frac{\sin(x-y)}{\cos(x+y)}\, dA$, where R is the triangular region enclosed by the lines $y = 0$, $y = x$, $x + y = \frac{\pi}{4}$.

3. Compute the Jacobian $\dfrac{\partial(x, y, z)}{\partial(\rho, \phi, \theta)}$ of the transformation corresponding to spherical coordinates $x = \rho\sin\phi\cos\theta$, $y = \rho\sin\phi\sin\theta$, $z = \rho\cos\phi$. This justifies your formula for $dV_{spherical}$.

4. Given a, b, c positive constants, the transformation $x = au$, $y = bv$, $z = cw$ can be rewritten as $x/a = u$, $y/b = v$, $z/c = w$, hence it maps the spherical region

$$u^2 + v^2 + w^2 \leq 1$$

into the ellipsoidal region

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} \leq 1\ . \tag{1}$$

(a) Use the above change of variables to find the volume of the solid ellipsoid (1).

(b) Use symmetry and the above change of variables to find the centroid of the upper-half of the ellipsoid (1) (the part of the ellipsoid above the xy-plane).