MAD 3301 - GRAPH THEORY – Spring 2022 – Review for Exam 1

Exam 1 covers all material we did from Chapters 1, 2, 3. I will keep the style of Prof. Ram's exams, so do use his past exams for practice. Any problem from the suggested assignment or from the worksheets could be an exam question. Additionally, be sure you also know:

KEY DEFINITIONS AND MAIN CONCEPTS

Graph, multi-graph, general graph (graph-like object), digraph, general digraph (digraph-like object), in-degree & out-degree (for digraphs), degree of a vertex, degree sequence, $\delta(G)$, $\Delta(G)$, sub-graphs, regular graphs, adjacency matrix; geometric, set-theoretic, and matrix representation of graphs; isomorphism of graphs; family of graphs (P_n, C_n, K_n,, bipartite graphs and complete bipartite graphs K_{m,n}, circulant graphs); walk, trail, circuit, cycle, path, distances in weighted or unweighted graphs; connected graphs, the connected components of a graph, weakly & strongly connected digraphs, bridge (cut-edge), edge connectivity, vertex connectivity, cut-vertex; trees, forests, non-identical trees, leaves, minimum spanning trees, rooted trees, levels, height of a tree, children, parent, n-ary trees, binary trees, binary coding, uniquely decipherable coding, weighted path-length of a binary coding, optimal binary coding.

MAIN ALGORITHMS

1. (a) Graphical sequence algorithm
   (b) Graph recovery algorithm,
2. (a) Breadth First Search (BFS) algorithm for finding distances in an unweighted graph.,
   (b) Dijkstra's distance algorithm in a weighted digraph or a weighted graph,
3. (a) Kruskal's minimum-weight spanning tree algorithm,
   (b) Prim's minimum-weight spanning tree algorithm,
4. (a) Prufer's tree-encoding algorithm,
   (b) Prufer's tree-decoding algorithm,
5. Huffman's optimal-coding algorithm.

MAIN THEOREMS – with proof, unless otherwise noted.

1. The decreasing sequence $<a,d_2,d_3, \ldots ,d_p>$ is graphical if and only if $<d_2-1,d_3-1,\ldots,d_{a+1}-1,d_{a+2},\ldots,d_p>$ is graphical . (*Graphical Sequence Theorem*)
   For proof only the direction $\rightarrow$ may be asked.
2. The number of walks of length n from v_i to v_j = (A^n) [i, j].
3. (a) A connected graph with p vertices has at least p-1 edges.
   (b) A graph with p vertices and more than (p-1)(p-2)/2 edges is always connected.
4. (a) If G is a disconnected graph, then G^c must be connected.
   (b) If G has p vertices and $\delta(G) \geq (p-1)/2$, then G is connected.
5. (a) Any tree with p vertices has exactly p-1 edges.
   (b) G is a tree if and only if there is exactly one path between any two vertices.
6. In any n-ary tree T with p vertices we have $\log_n [\{p.(n-1) +1\}/n] \leq h(T) \leq p-1$.
7. There are p^{p-2} different (non-identical) trees on p distinct vertices. (*Cayley's theorem*)
   Proof of Cayley's theorem will not be asked