MAD 3301 - GRAPH THEORY – Spring 2022 – Review for Exam 2

Exam 2 covers all material we did from Chapters 6, 7 + the 4/07 lecture on graph coloring. I will keep the style of Prof. Ram's exams, so do use his past exams for practice, but adjust for the difference in coverage. Any problem from the suggested assignment or from the worksheets could be an exam question. Additionally, be sure you also know:

KEY CONCEPTS AND MAIN DEFINITIONS:

Euler circuits, Open Euler trails, Chinese postman problem, minimum postman walk, Hamilton cycles, Hamilton paths, Ore-type graphs, traveling salesman problem, planar graphs, planar embeddings, maximal planar graphs, K_5, $K_{3,3}$, creating & merging out vertices of degree 2, graph homeomorphisms, pieces of a subgraph, segments, spheroidal graphs, stereographical projection, polyhedral graphs, the five regular polyhedra, crossing number of a graph, toroidal graphs, geometric dual of a graph, legal colorings, chromatic number.

MAIN ALGORITHMS:

1. (a) Fleury's Euler-circuit (& open Euler-trail) algorithm.
   (b) Minimal postman-walk algorithm (Chinese postman algorithm).
2. The basic rules for seeking a Hamiltonian cycle (or showing there is none).
3. Pre-processing graphs for planarity & the DMP planarity algorithm.
4. The basic coloring algorithm (which guarantees a coloring with $\Delta(G) + 1$ colors)

MAIN THEOREMS:

1. (a) The connected graph G has an Euler circuit iff each vertex in G is of even degree.
   (b) It has an open Euler trail iff G has exactly two vertices of odd degree. (*Euler's Theorem*)
3. (a) If $\deg(x) + \deg(y) \geq p$ for all pairs of non-adjacent vertices x & y in G and $p \geq 3$, then G has a Hamilton cycle. (b) If $\deg(x) + \deg(y) \geq p-1$ for all pairs of non-adjacent vertices x & y in G, then G has a Hamilton path. (*Ore's Theorem*)
4. (a) If G is a connected planar graph, then $r = q+2-p$. (*Euler's formula*)
   (b) If G is a planar graph with k components, then $r = q+k+1-p$. (*Gen. Euler's formula*)
5. (a) In any planar graph with $p \geq 3$ vertices, we have $q \leq 3p-6$, with equality iff all regions are bounded by three edges.
   (b) Let G be a planar graph with $p \geq 3$ vertices. TFAE: (i) G is maximal planar; (ii) $q = 3p-6$; (iii) all regions are bounded by three edges.
   (c) Let G be a planar graph with $p \geq 3$ vertices and assume *girth*(G) = $k \geq 3$. Then $q \leq k(p-2)/(k-2)$.
6. G is planar if an only if G has no subgraph which is homeomorphic to K_5 or $K_{3,3}$. (*Kuratowski's Theorem*) – know the statement and the proof for the easy direction only.
7. The 5 regular polyhedra Theorem.
8. If a graph G has crossing number *cr*(G) = 1 then it is toroidal (but converse is not true).
9. (a) Let G be a graph with at least one edge. Then G is bipartite iff $\chi(G) = 2$ iff G contains no cycle of odd length.
   (b) In any graph G, $\chi(G) \leq \Delta(G) + 1$. (Here $\Delta(G)$ = largest degree in G.)