(1) <u>Algorithm for finding a shortest cycle from a given vertex</u>

<u>Input</u> : A connected graph $G=(V,E)$ with distinguished vertex x.

<u>Output</u> : The length of a shortest cycle starting from x and one such cycle C (if such cycle exists)

<u>Method</u> : A BFS type of search starting from x.

Each visited vertex v will be labeled (v, i, p) where i is the step in the process (or distance from x) and p is the parent of v in the BFS tree

In the process, U denotes the set of <u>unlabeled</u> vertices at that stage.

We'll also use a procedure BTA (back track ancestor), so that $BTA((v,i,p),(u,j,\ell))$ gives the first common ancestor a of vertices v and u in the BFS tree and the paths P_1 from a to v and P_2 from a to u

It is an exercise for you to write the algorithm for the BTA procedure

In the steps (4) and (6) of the algorithm, we may have to erase certain edges from the given graph, thus the set E is allowed to change

<u>More notations</u> : If P is a path, then $\overline{P}$ denotes the path reversing the order

For a vertex v, $N(v)$ denotes the set of vertices adjacent to v (the neighborhood of v.

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Steps of the algorithm:

$\emptyset$ as x is the root of the DFS tree so x has no parent

(1) If $|N(x)| \leq 1$ then STOP and say NO CYCLE from x;

(2) Let $i = 0$; label x with $(x, 0, \emptyset)$; let $U = V \setminus \{x\}$;

(3) Let $i = 1$; label each $y \in N(x)$ as $(y, 1, x)$; let $U = U \setminus N(x)$;

(4) If there are no adjacent vertices with label i then go to step (5)

Else for (v, i, p) and (u, i, q) with $vu \in E$

If $BTA((v,i,p),(u,i,q)) = x$ then STOP and say CYCLE of length $2i+1$, $C = P_1 \overline{P_2}$

Else let $E = E \setminus \{vu\}$ ~~and goto~~

Go to Step (4)

(5) If $U = \emptyset$ then STOP and say NO CYCLE from x;

(6) If there are no two vertices v, u with label i, $v \neq u$, so that $N(v) \cap N(u) \cap U \neq \emptyset$ then go to step (7)

Else

For (v,i,p) and (u,i,q) with $N(v) \cap N(u) \cap U \neq \emptyset$

If $BTA((v,i,p),(u,i,q)) = x$ then STOP and say CYCLE of length $2(i+1)$, $C = P_1 w \overline{P_2}$ for some $w \in N(v) \cap N(u) \cap U$

Else for all $w \in N(v) \cap N(u) \cap U$, let $E = E \setminus \{vw\}$

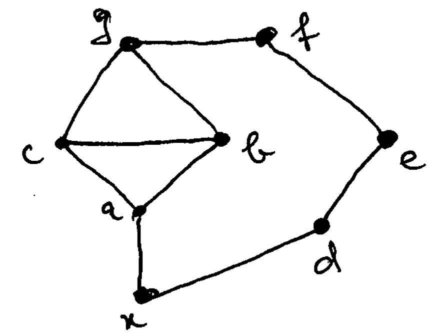
Go to Step (6)

(7) For every vertex $w \in U$ adjacent to a vertex p with label i, label $(w, i+1, p)$ and let $U = U \setminus \{w\}$;

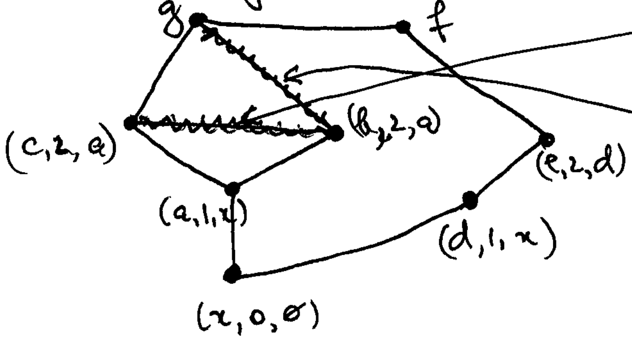
(8) Let $i \to i+1$ and go to Step (4).

(3) Here is an example of the algorithm.

Let G be the graph below, with x the distinguished vertex



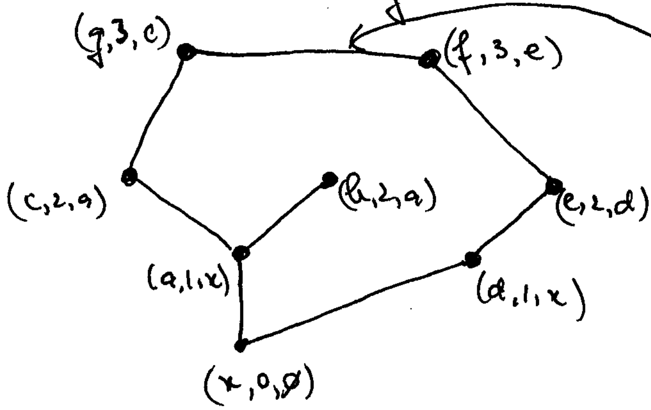
The application of the algorithm produces the following labeling and elimination of certain edges at the stage $i=2$



eliminated in step (4) as $BTA((c,2,a),(b,2,a)) = a \neq x$

eliminated in step (6) as $DTA((c,2,a),(b,2,a)) = a \neq x$

Once i becomes 3 in step (8) and we go back to step (4), we'll have the labeling



and in step (4) cycle of length $2 \cdot 3 + 1 = 7$ will be detected, as $BTA((g,3,c),(f,3,e)) = x$ and the cycle is

$$C = \underbrace{x\,a\,c\,g}_{P_1}\ \underbrace{f\,e\,d\,x}_{\overline{P_2}}$$