Name: _____ PantherID: _____

Exam 1 – MAD 3301 Graph Theory – Spring 2022

Important Rules:

A. Any electronic device (cell phone, calculator of any kind, smart-watch, etc.) should be turned off at the beginning of the exam and placed in your bag, NOT in your pocket. Electronic items, notes, texts, or formula sheets should NOT be used at any time during the examination. Concentrate on your own exam. Do not look at your neighbor's paper or try to communicate with your neighbor.

B. Unless otherwise mentioned, to receive full credit you must show your work. Answers which are not supported by work might receive no credit.

1. (12 pts) Find, if possible, a graph with degree sequence $< 4, 4, 3, 3, 2, 2 >$ by using the *Graphical Sequence Algorithm.*

2. (10 pts) For each of the following circle True or False questions. No justification is necessary. (2 pts each)

(a) For any vertex v in a digraph G, $indeg(v) = outdeg(v)$. **True** **False**

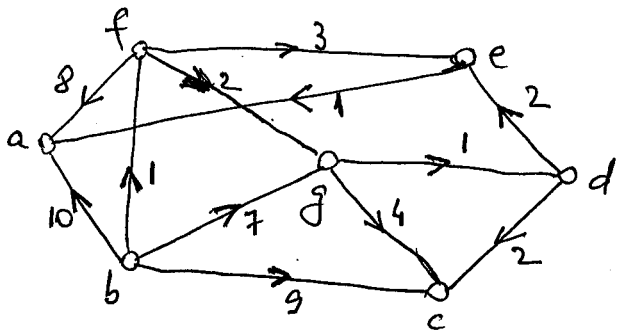
(b) $K_{2,3}$ and $K_{3,2}$ are isomorphic. **True** **False**

(c) $K_{3,3}$ contains a subgraph isomorphic to K_3. **True** **False**

(d) The number of non-identical trees on 10 vertices is 10^8. **True** **False**

(e) If a graph G contains a circuit, then G contains a cycle. **True** **False**

3. (12 pts) Find the distances from vertex b to each of the other vertices of the weighted digraph on the right by using *Dijkstra's Algorithm.*



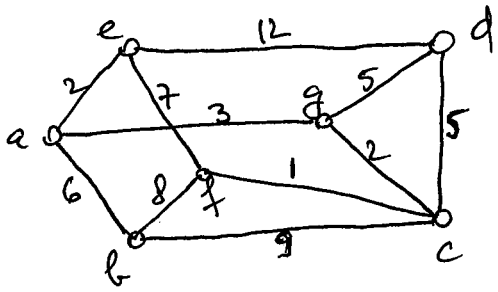
4. (12 pts) Give a precise definition to each of the following notions:

(a) A *connected component* H of a connected graph G;

(b) The *height* $h(T)$ of a rooted tree (T, v), where v is the root;

(c) The *vertex connectivity number* $k_v(G)$ of a graph G.

5. (12 pts) For the graph on the right apply Prim's algorithm starting at vertex d to obtain a minimal weight spanning tree.



6. (12 pts) Find the tree that corresponds to the sequence $< 5, 1, 5, 3 >$ via Prüfer's Tree Decoding Algorithm.

7. (12 pts) The five characters *a, b, c, d, e* occur with frequencies 25, 10, 15, 15, 35 respectively. Find an optimal binary coding for these five characters and find the weighted-path length of your coding by using Huffman's algorithm. (4 pts bonus) How many different optimal codings are possible in this case?

8. (24 pts) Choose TWO of the following THREE proofs. If you do all three, only top two scores will count while the third score may give you some bonus towards an earlier problem with a lower score.

(A) (12 pts) Let G be a graph and G^c be the complement of G. Prove that if G is a disconnected graph, then G^c must be a connected graph.

(B) (12 pts) Use induction to prove that in any tree with p vertices the number of edges is $p-1$.

(C) (12 pts) Show that if G is a connected graph which contains no vertices of odd degree, then G contains no bridges.