Name: ______________________ PantherID: ______________________

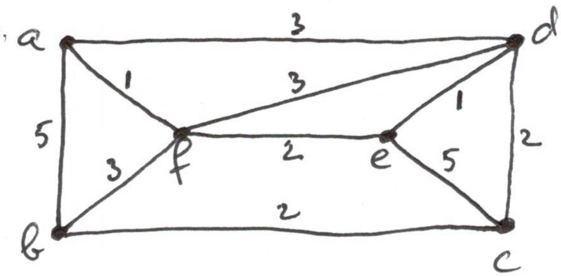
Exam 2 – MAD 3301 Graph Theory – Spring 2022

Important Rules:

A. Any electronic device (cell phone, calculator of any kind, smart-watch, etc.) should be turned off at the beginning of the exam and placed in your bag, NOT in your pocket. Electronic items, notes, texts, or formula sheets should NOT be used at any time during the examination. Concentrate on your own exam. Do not look at your neighbor's paper or try to communicate with your neighbor.

B. Unless otherwise mentioned, to receive full credit you must show your work. Answers which are not supported by work might receive no credit.

1. (15 pts) Find a *minimum postman walk* for the graph on the right, using the *Postman Algorithm.* Also find the *total length* of your minimum postman walk. You can assume that the postman must start and return at vertex a.



2. (15 pts) For each of the following statements circle True or False. No justification is necessary. (3 pts each)

(a) K_n is planar for any $n \geq 3$. **True** **False**

(b) K_n has a Hamiltonian cycle for any $n \geq 3$. **True** **False**

(c) $K_{6,8}$ has an Euler circuit. **True** **False**

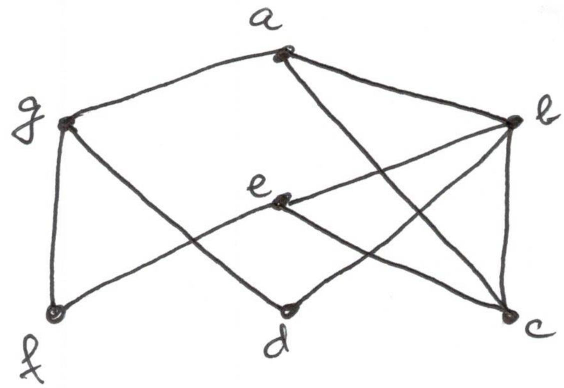
(d) Any graph G that is toroidal is also spheroidal. **True** **False**

(e) There is no maximal planar graph G with $q = 20$ edges. **True** **False**

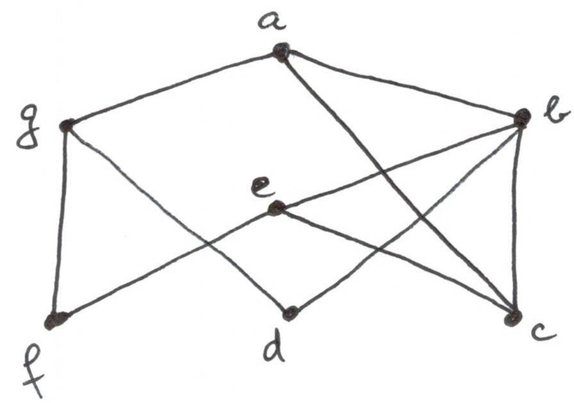
3. (15 pts) Prove that in any graph G, $\chi(G) \leq \Delta(G) + 1$.

4. (15 pts) Determine whether or not the graph G on the right is planar, by using the *DMP Planarity algorithm.* Show the embeddings for each step of the algorithm.

(5 pts bonus) Is the graph G toroidal? Briefly justify.



5. (15 pts) Both parts of this problem refer to the graph G on the right.



(a) (7 pts) Define an (open) Euler trail and determine if the graph G on the right admits an Euler trail. Justify.

(b) (8 pts) Find, with justification, $\chi(G)$.

6. (15 pts) Let G be a planar graph with p vertices and q edges and let $\text{girth}(G) = k \geq 3$.

Show that $q \leq \dfrac{k(p-2)}{k-2}$. When does equality hold?

7. (15 pts) Choose ONE of the following TWO. If you do both, only the top score will count, while the second score may give you some bonus towards an earlier problem with a lower score.

(A) State Ore's Theorem and then use it to prove the following: Let G be a connected graph with $p \geq 3$ vertices. If for any pair of non-adjacent vertices x, y, $\deg(x) + \deg(y) \geq p - 1$, then G has a Hamiltonian path

(B) A soccer ball can be thought of as a convex polyhedron in which all vertices have degree 3, and two regular hexagons and one regular pentagon are incident at each vertex. Find, with mathematical justification, the number of pentagon faces and the number of hexagon faces of a soccer ball.