Names: _____   PantherIDs: _____

Homework 1 - Graph Theory - Spring '22   **Due Thursday, Jan. 18**

1. Use the graphical sequence algorithm to determine if each of the following sequences are graphical, and, in case the answer is affirmative, find a simple graph with the corresponding degree sequence.

(a) 3, 3, 3, 1

(b) 4, 3, 3, 2, 2

2. Show that in any group of people there are (at least) two with the same number of friends (in the same group of people). What is the Graph Theory interpretation of the problem?

3. (a) By the method indicated in our first class meeting (that is, using as many loops as possible), it follows that any decreasing sequence of non-negative integers $S = < d_1 \geq d_2 \geq \ ... \ \geq d_p >$ with even sum can be the degree sequence of a general graph with single edges and with (possibly multiple) loops. Based on the class method, write an algorithm to construct a general graph with a given a degree sequence S with even sum.

(b) Here is the equivalent problem for a *multi-graph*, that is, a general graph in which multi-edges are allowed, but loops are NOT allowed. Show that a decreasing sequence of non-negative integers $S = < d_1 \geq d_2 \geq \ ... \ \geq d_p >$ is the degree sequence of a multi-graph if and only if

$$\sum_{k=1}^{p} d_k \ \text{ is even, and } \ \ d_1 \leq \sum_{k=2}^{p} d_k$$