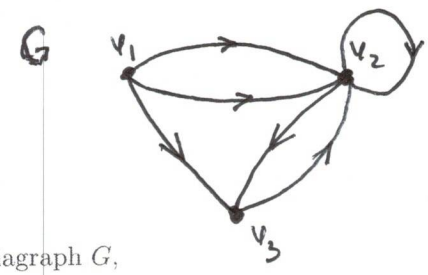
Worksheet 1-27 Graph Theory – Spring '22 **Names:** _____

Due ~~Tuesday, 1/31~~. Thursday, Feb. 3

1. Consider the general digraph G given below:



Let ~~also~~ $\tilde{G}$ be the *supporting general graph* of the general diagraph G, that is, $\tilde{G}$ is obtained from G by just erasing the arrows on all edges (make them all undirected).

(a) Draw $\tilde{G}$.

(b) Write the adjacency matrices, A_G and $A_{\tilde{G}}$ and show that $A_{\tilde{G}} = A_G + (A_G)^T$.
Does this hold for any general digraph G with supporting graph $\tilde{G}$?
Briefly explain your answer.

(c) For the given digraph G, use the class theorem to find the number of walks of length 3 from v_1 to v_3.

(d) Fill in one distance table for G and one distance table for $\tilde{G}$. (Do it directly, no need to apply BFS algorithm.)

2. Show that for a general graph G, the graph distance d, as defined, satisfies all properties of a distance:

(i) $d(x, y) \geq 0$, with $d(x, y) = 0$ if and only if $x = y$;

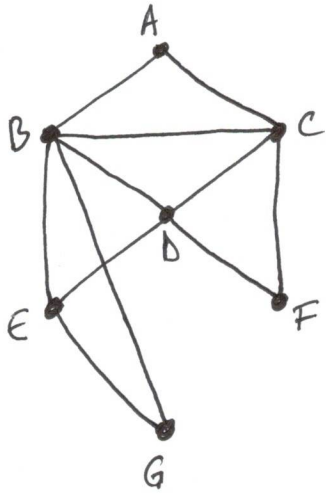
(ii) $d(x, y) = d(y, x)$, for any vertices x, y.

(iii) $d(x, z) \leq d(x, y) + d(y, z)$, for any vertices x, y, z.

Note: In the case of a general digraph, property (ii) may fail, as Pb. 1(d) shows.

3. (a) For the BFS class example, draw a search tree and the corresponding labels starting at vertex C.

(b) Different answers (all correct) are possible for part (a). Briefly explain why.



4. Modify the BFS algorithm to find the length of the smallest cycle (if any) starting at a given vertex x of a graph G. What if you are also asked to list that cycle?

Note: Especially for Computer Science students, you may solve problem 4 by actually writing a program and showing me the code and a couple examples on how it works.

Important Final Note: Again, I strongly encourage you to form teams and work together on these assignments. I may give some bonus when I see good team work with contributions from different team members.