Names: _____ **PantherIDs:** _____

Worksheet 3-24 - Graph Theory

1. (a) Show that if G is a connected simple planar graph with $girth(G) = k$, $k \geq 3$, then

$$q \leq \frac{k}{k-2}(p-2) \ ,$$

where, as usual, $p = |V(G)|$, $q = |E(G)|$.

(b) Use part (a) to show that $K_{3,3}$ is not planar.

(c) Use part (a) to show that the Petersen graph is not planar.

2. A graph G is said to be *n-partite* graph if it is possible to partition the vertex set of G into n nonempty subsets such that any edge of G joins two vertices in different partite sets. (In the case $n = 2$, we have the usual notion of a bipartite graph.) Complete n-partite graphs have all possible edges between the partite sets. We denote by $K_{p_1,p_2,...,p_n}$ the complete n-partite graph with partite sets of order $p_1, p_2, ..., p_n$.

(a) Show that $K_{2,2,2}$ is planar by providing a planar embedding (please use colors for vertices in the partition).

(b) Show that $K_{2,2,3}$ is NOT planar.

3. Show that the Petersen graph contains a subgraph which is a subdivision of $K_{3,3}$. Thus, via Kuratowski's Theorem (the easy part), this gives another proof that the Petersen graph is not planar.