Worksheet 3/31

Chapter 6

Pb.1. Show that

$$cr(circ(7;\{1,2\})) = 1.$$

Recall that $circ(n;S)$ is the circulant graph on n vertices associated to the set S, and $cr(G)$ denotes the crossing number of the graph G.

Pb.2 : Pb. 8, chp.6, textbook

Pb.3 : Pb.16, chp. 6, textbook

Suggested exercises for Chp.6: # 1, 2, 5, 7–12 all, 14, 16

Exercises for Chapter 6.

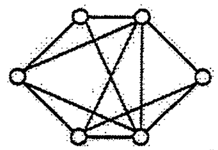
1. Show that if G is a plane (p, q) graph with r regions, then $p - q + r = 1 + k(G)$ where $k(G)$ is the number of components of G.

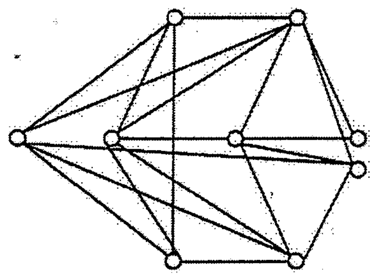
Exercises for Ch. 6

2. An (f, d)–*regular polyhedron graph* is a plane graph that is d-regular ($d \geq 3$) and each of its faces has f sides. Use Euler's formula to show that there are only five regular polyhedron graphs. (Hint: The dodecahedron is a $(5, 3)$–regular polyhedron graph.)
3. Show that "homeomorphic with" is an equivalence relation on the set of graphs.
4. Show that a graph is planar if, and only if, each of its blocks (maximal 2-connected subgraphs) is planar.
5. Let G be a maximal planar graph of order $p \geq 4$. Also let p_i denote the number of vertices of degree i in G, where $i = 3, 4, \ldots, \Delta(G) = n$. Show that
$$3p_3 + 2p_4 + p_5 = p_7 + 2p_8 + \cdots + (n-6)p_n + 12.$$
6. Show that any maximal planar (p, q) graph contains a bipartite subgraph with $2\frac{q}{3}$ edges.
7. Find an example of a planar graph that contains no vertex of degree less than 5.
8. Prove that every planar graph of order $p \geq 4$ contains at least four vertices of degree at most 5.
9. Show that the Petersen graph (Figure 7.3.2) contains a subgraph homeomorphic with $K_{3,3}$ and is therefore not planar.
10. Show that if G is a connected planar (p, q) graph with girth (shortest cycle length) $g(G) = k \geq 3$, then $|E| \leq \frac{k(p-2)}{(k-2)}$.
11. Use the last result to again show that the Petersen graph is not planar.
12. A graph is self-dual if it is isomorphic to its own geometric dual. Show that if G is self-dual, then $2|V| = |E| + 2$. Further, show that not every graph with this property is self-dual.
13. If G is a connected plane graph with spanning tree T and $E^* = \{e^* \in E(G^*) \mid e \notin E(T)\}$, show that $T^* = \langle E^* \rangle$ is a spanning tree of G^*.
14. Show that if $|V(G)| \geq 11$, then at least one of G and $\overline{G}$ is nonplanar.
15. Show that the average degree in a planar graph is actually less than 6. (Note that this provides an alternate proof to Corollary 6.1.2.)

Exercises for Ch. 6

16. Use the DMP algorithm to test the planarity of the following graphs.





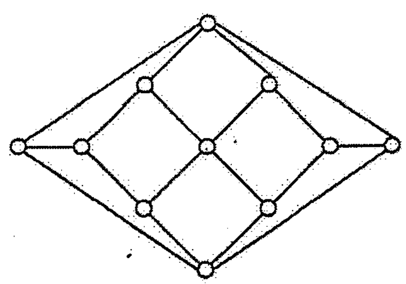
17. Prove path properties 1–5 relating to the Hopcroft-Tarjan planarity algorithm.

18. Use the Hopcroft-Tarjan planarity algorithm to test the planarity of the graphs from exercise 16 in Chapter 6.

19. Prove Theorem 6.4.1.

20. How might you actually keep track of the paths both inside and outside of a given cycle? How much information must actually be recorded?

21. Use Grinberg's theorem to show that the graph below is not hamiltonian.



Exercises for Ch. 6

22. Show that no hamiltonian cycle in the graph below can contain both of the edges e and f.