

**Due Wednesday, June 19.**

1. (20 pts) Consider the Cat and Mouse game presented in class (see also the link on the website). This time assume that there are 4 boxes arranged on a circle and labeled 1,2,3,4, in this order (thus the neighbors of box 1 are boxes 2 and 4). At every stage of the game (every day) the cat and the mouse jump from their boxes and land, randomly, with probability  $1/3$  either in the box where they left or in one of the neighboring boxes. The game ends when the cat and the mouse land in the same box and the cat eats the mouse. Suppose that initially the cat is in box 1 and the mouse is in box 3.

Represent the game as a Markov Chain (determine the states, the probability matrix, eigenvalues, eigenvectors, etc.) and then find the smallest  $n$  so that after  $n$  days the mouse has a probability of less than 1 percent of being alive.