Review of Algebra

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Here we review the basic rules and procedures of algebra that you need to know in order to be successful in calculus.

Arithmetic Operations

The real numbers have the following properties:

$a + b = b + a$	$ab = ba$	(Commutative Law)
$(a + b) + c = a + (b + c)$	$(ab)c = a(bc)$	(Associative Law)
$a(b + c) = ab + ac$		(Distributive law)

In particular, putting $a = -1$ in the Distributive Law, we get

$$-(b + c) = (-1)(b + c) = (-1)b + (-1)c$$

and so

$$-(b + c) = -b - c$$

EXAMPLE 1

(a) $(3xy)(-4x) = 3(-4)x^2y = -12x^2y$

(b) $2t(7x + 2tx - 11) = 14tx + 4t^2x - 22t$

(c) $4 - 3(x - 2) = 4 - 3x + 6 = 10 - 3x$

If we use the Distributive Law three times, we get

$$(a + b)(c + d) = (a + b)c + (a + b)d = ac + bc + ad + bd$$

This says that we multiply two factors by multiplying each term in one factor by each term in the other factor and adding the products. Schematically, we have

$$(a + b)(c + d)$$

In the case where $c = a$ and $d = b$, we have

$$(a + b)^2 = a^2 + ba + ab + b^2$$

or

1 $$(a + b)^2 = a^2 + 2ab + b^2$$

Similarly, we obtain

2 $$(a - b)^2 = a^2 - 2ab + b^2$$

EXAMPLE 2

(a) $(2x + 1)(3x - 5) = 6x^2 + 3x - 10x - 5 = 6x^2 - 7x - 5$

(b) $(x + 6)^2 = x^2 + 12x + 36$

(c) $$\begin{aligned} 3(x - 1)(4x + 3) - 2(x + 6) &= 3(4x^2 - x - 3) - 2x - 12 \\ &= 12x^2 - 3x - 9 - 2x - 12 \\ &= 12x^2 - 5x - 21 \end{aligned}$$

Fractions

To add two fractions with the same denominator, we use the Distributive Law:

$$\frac{a}{b} + \frac{c}{b} = \frac{1}{b} \times a + \frac{1}{b} \times c = \frac{1}{b}(a + c) = \frac{a + c}{b}$$

Thus, it is true that

$$\frac{a + c}{b} = \frac{a}{b} + \frac{c}{b}$$

But remember to avoid the following common error:

$$\frac{a}{b + c} \neq \frac{a}{b} + \frac{a}{c}$$

(For instance, take $a = b = c = 1$ to see the error.)

To add two fractions with different denominators, we use a common denominator:

$$\frac{a}{b} + \frac{c}{d} = \frac{ad + bc}{bd}$$

We multiply such fractions as follows:

$$\frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd}$$

In particular, it is true that

$$\frac{-a}{b} = -\frac{a}{b} = \frac{a}{-b}$$

To divide two fractions, we invert and multiply:

$$\frac{\frac{a}{b}}{\frac{c}{d}} = \frac{a}{b} \times \frac{d}{c} = \frac{ad}{bc}$$

EXAMPLE 3

(a) $\dfrac{x+3}{x} = \dfrac{x}{x} + \dfrac{3}{x} = 1 + \dfrac{3}{x}$

(b) $$\frac{3}{x-1} + \frac{x}{x+2} = \frac{3(x+2) + x(x-1)}{(x-1)(x+2)} = \frac{3x + 6 + x^2 - x}{x^2 + x - 2}$$
$$= \frac{x^2 + 2x + 6}{x^2 + x - 2}$$

(c) $\dfrac{s^2t}{u} \cdot \dfrac{ut}{-2} = \dfrac{s^2t^2u}{-2u} = -\dfrac{s^2t^2}{2}$

(d) $$\frac{\dfrac{x}{y} + 1}{1 - \dfrac{y}{x}} = \frac{\dfrac{x+y}{y}}{\dfrac{x-y}{x}} = \frac{x+y}{y} \times \frac{x}{x-y} = \frac{x(x+y)}{y(x-y)} = \frac{x^2 + xy}{xy - y^2}$$ ■

Factoring

We have used the Distributive Law to expand certain algebraic expressions. We sometimes need to reverse this process (again using the Distributive Law) by factoring an expression as a product of simpler ones. The easiest situation occurs when the expression has a common factor as follows:

Expanding ⟶

$$3x(x-2) = 3x^2 - 6x$$

⟵ Factoring

To factor a quadratic of the form $x^2 + bx + c$ we note that

$$(x + r)(x + s) = x^2 + (r + s)x + rs$$

so we need to choose numbers r and s so that $r + s = b$ and $rs = c$.

EXAMPLE 4 Factor $x^2 + 5x - 24$.

SOLUTION The two integers that add to give 5 and multiply to give -24 are -3 and 8. Therefore

$$x^2 + 5x - 24 = (x - 3)(x + 8)$$ ■

EXAMPLE 5 Factor $2x^2 - 7x - 4$.

SOLUTION Even though the coefficient of x^2 is not 1, we can still look for factors of the form $2x + r$ and $x + s$, where $rs = -4$. Experimentation reveals that

$$2x^2 - 7x - 4 = (2x + 1)(x - 4)$$ ■

Some special quadratics can be factored by using Equations 1 or 2 (from right to left) or by using the formula for a difference of squares:

3 $$a^2 - b^2 = (a - b)(a + b)$$

The analogous formula for a difference of cubes is

4 $$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$$

which you can verify by expanding the right side. For a sum of cubes we have

5 $$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$$

EXAMPLE 6

(a) $x^2 - 6x + 9 = (x - 3)^2$ (Equation 2; $a = x, b = 3$)

(b) $4x^2 - 25 = (2x - 5)(2x + 5)$ (Equation 3; $a = 2x, b = 5$)

(c) $x^3 + 8 = (x + 2)(x^2 - 2x + 4)$ (Equation 5; $a = x, b = 2$)

EXAMPLE 7 Simplify $\dfrac{x^2 - 16}{x^2 - 2x - 8}$.

SOLUTION Factoring numerator and denominator, we have

$$\frac{x^2 - 16}{x^2 - 2x - 8} = \frac{(x - 4)(x + 4)}{(x - 4)(x + 2)} = \frac{x + 4}{x + 2}$$

To factor polynomials of degree 3 or more, we sometimes use the following fact.

6 **The Factor Theorem** If P is a polynomial and $P(b) = 0$, then $x - b$ is a factor of $P(x)$.

EXAMPLE 8 Factor $x^3 - 3x^2 - 10x + 24$.

SOLUTION Let $P(x) = x^3 - 3x^2 - 10x + 24$. If $P(b) = 0$, where b is an integer, then b is a factor of 24. Thus, the possibilities for b are $\pm1, \pm2, \pm3, \pm4, \pm6, \pm8, \pm12$, and ±24. We find that $P(1) = 12$, $P(-1) = 30$, $P(2) = 0$. By the Factor Theorem, $x - 2$ is a factor. Instead of substituting further, we use long division as follows:

$$\begin{array}{r@{}l} & x^2 - x - 12 \\ x - 2\,\big)\! & x^3 - 3x^2 - 10x + 24 \\ & x^3 - 2x^2 \\ & \quad -x^2 - 10x \\ & \quad -x^2 + 2x \\ & \qquad -12x + 24 \\ & \qquad -12x + 24 \end{array}$$

Therefore $$\begin{aligned} x^3 - 3x^2 - 10x + 24 &= (x - 2)(x^2 - x - 12) \\ &= (x - 2)(x + 3)(x - 4) \end{aligned}$$

Completing the Square

Completing the square is a useful technique for graphing parabolas or integrating rational functions. Completing the square means rewriting a quadratic $ax^2 + bx + c$

in the form $a(x + p)^2 + q$ and can be accomplished by:

1. Factoring the number a from the terms involving x.
2. Adding and subtracting the square of half the coefficient of x.

In general, we have

$$\begin{aligned} ax^2 + bx + c &= a\left[x^2 + \frac{b}{a}x\right] + c \\ &= a\left[x^2 + \frac{b}{a}x + \left(\frac{b}{2a}\right)^2 - \left(\frac{b}{2a}\right)^2\right] + c \\ &= a\left(x + \frac{b}{2a}\right)^2 + \left(c - \frac{b^2}{4a}\right) \end{aligned}$$

EXAMPLE 9 Rewrite $x^2 + x + 1$ by completing the square.

SOLUTION The square of half the coefficient of x is $\frac{1}{4}$. Thus

$$x^2 + x + 1 = x^2 + x + \tfrac{1}{4} - \tfrac{1}{4} + 1 = \left(x + \tfrac{1}{2}\right)^2 + \tfrac{3}{4}$$ ■

EXAMPLE 10

$$\begin{aligned} 2x^2 - 12x + 11 = 2[x^2 - 6x] + 11 &= 2[x^2 - 6x + 9 - 9] + 11 \\ &= 2[(x - 3)^2 - 9] + 11 = 2(x - 3)^2 - 7 \end{aligned}$$ ■

Quadratic Formula

By completing the square as above we can obtain the following formula for the roots of a quadratic equation.

7 The Quadratic Formula The roots of the quadratic equation $ax^2 + bx + c = 0$ are

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

EXAMPLE 11 Solve the equation $5x^2 + 3x - 3 = 0$.

SOLUTION With $a = 5$, $b = 3$, $c = -3$, the quadratic formula gives the solutions

$$x = \frac{-3 \pm \sqrt{3^2 - 4(5)(-3)}}{2(5)} = \frac{-3 \pm \sqrt{69}}{10}$$ ■

The quantity $b^2 - 4ac$ that appears in the quadratic formula is called the **discriminant**. There are three possibilities:

1. If $b^2 - 4ac > 0$, the equation has two real roots.
2. If $b^2 - 4ac = 0$, the roots are equal.
3. If $b^2 - 4ac < 0$, the equation has no real root. (The roots are complex.)

These three cases correspond to the fact that the number of times the parabola $y = ax^2 + bx + c$ crosses the x-axis is 2, 1, or 0 (see Figure 1). In case (3) the quadratic $ax^2 + bx + c$ can't be factored and is called **irreducible.**

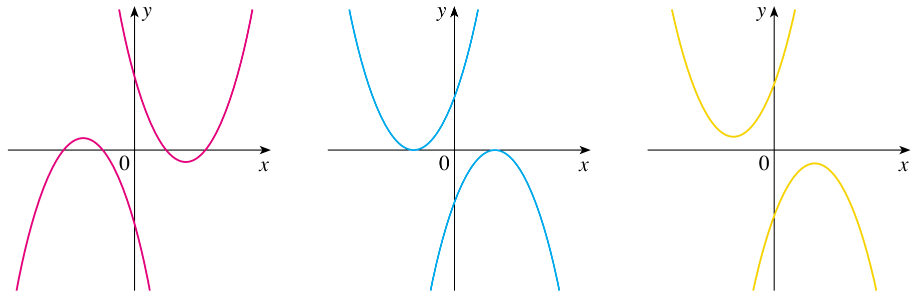

FIGURE 1
Possible graphs of $y = ax^2 + bx + c$

(a) $b^2 - 4ac > 0$ (b) $b^2 - 4ac = 0$ (c) $b^2 - 4ac < 0$

EXAMPLE 12 The quadratic $x^2 + x + 2$ is irreducible because its discriminant is negative:

$$b^2 - 4ac = 1^2 - 4(1)(2) = -7 < 0$$

Therefore, it is impossible to factor $x^2 + x + 2$.

The Binomial Theorem

Recall the binomial expression from Equation 1:

$$(a + b)^2 = a^2 + 2ab + b^2$$

If we multiply both sides by $(a + b)$ and simplify, we get the binomial expansion

8 $$(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$$

Repeating this procedure, we get

$$(a + b)^4 = a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4$$

In general, we have the following formula.

9 **The Binomial Theorem** If k is a positive integer, then

$$(a + b)^k = a^k + ka^{k-1}b + \frac{k(k-1)}{1 \cdot 2}a^{k-2}b^2$$
$$+ \frac{k(k-1)(k-2)}{1 \cdot 2 \cdot 3}a^{k-3}b^3$$
$$+ \cdots + \frac{k(k-1)\cdots(k-n+1)}{1 \cdot 2 \cdot 3 \cdot \cdots \cdot n}a^{k-n}b^n$$
$$+ \cdots + kab^{k-1} + b^k$$

EXAMPLE 13 Expand $(x - 2)^5$.

SOLUTION Using the Binomial Theorem with $a = x$, $b = -2$, $k = 5$, we have

$$(x - 2)^5 = x^5 + 5x^4(-2) + \frac{5 \cdot 4}{1 \cdot 2} x^3(-2)^2 + \frac{5 \cdot 4 \cdot 3}{1 \cdot 2 \cdot 3} x^2(-2)^3 + 5x(-2)^4 + (-2)^5$$

$$= x^5 - 10x^4 + 40x^3 - 80x^2 + 80x - 32$$

Radicals

The most commonly occurring radicals are square roots. The symbol $\sqrt{}$ means "the positive square root of." Thus

$$x = \sqrt{a} \qquad \text{means} \qquad x^2 = a \quad \text{and} \quad x \geqslant 0$$

Since $a = x^2 \geqslant 0$, the symbol $\sqrt{a}$ makes sense only when $a \geqslant 0$. Here are two rules for working with square roots:

10
$$\sqrt{ab} = \sqrt{a}\,\sqrt{b} \qquad \sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$$

However, there is no similar rule for the square root of a sum. In fact, you should remember to avoid the following common error:

$$\sqrt{a + b} \neq \sqrt{a} + \sqrt{b}$$

(For instance, take $a = 9$ and $b = 16$ to see the error.)

EXAMPLE 14

(a) $\dfrac{\sqrt{18}}{\sqrt{2}} = \sqrt{\dfrac{18}{2}} = \sqrt{9} = 3$

(b) $\sqrt{x^2y} = \sqrt{x^2}\,\sqrt{y} = |x|\sqrt{y}$

Notice that $\sqrt{x^2} = |x|$ because $\sqrt{}$ indicates the positive square root. (See Appendix A.)

In general, if n is a positive integer,

$$x = \sqrt[n]{a} \qquad \text{means} \qquad x^n = a$$

If n is even, then $a \geqslant 0$ and $x \geqslant 0$.

Thus $\sqrt[3]{-8} = -2$ because $(-2)^3 = -8$, but $\sqrt[4]{-8}$ and $\sqrt[6]{-8}$ are not defined. The following rules are valid:

$$\sqrt[n]{ab} = \sqrt[n]{a}\,\sqrt[n]{b} \qquad \sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$$

EXAMPLE 15 $\sqrt[3]{x^4} = \sqrt[3]{x^3x} = \sqrt[3]{x^3}\,\sqrt[3]{x} = x\sqrt[3]{x}$

To **rationalize** a numerator or denominator that contains an expression such as $\sqrt{a} - \sqrt{b}$, we multiply both the numerator and the denominator by the conjugate radical $\sqrt{a} + \sqrt{b}$. Then we can take advantage of the formula for a difference of squares:

$$\left(\sqrt{a} - \sqrt{b}\right)\left(\sqrt{a} + \sqrt{b}\right) = \left(\sqrt{a}\right)^2 - \left(\sqrt{b}\right)^2 = a - b$$

EXAMPLE 16 Rationalize the numerator in the expression $\dfrac{\sqrt{x+4} - 2}{x}$.

SOLUTION We multiply the numerator and the denominator by the conjugate radical $\sqrt{x+4} + 2$:

$$\frac{\sqrt{x+4} - 2}{x} = \left(\frac{\sqrt{x+4} - 2}{x}\right)\left(\frac{\sqrt{x+4} + 2}{\sqrt{x+4} + 2}\right) = \frac{(x+4) - 4}{x\left(\sqrt{x+4} + 2\right)}$$

$$= \frac{x}{x\left(\sqrt{x+4} + 2\right)} = \frac{1}{\sqrt{x+4} + 2}$$

Exponents

Let a be any positive number and let n be a positive integer. Then, by definition,

1. $a^n = \underbrace{a \cdot a \cdot \cdots \cdot a}_{n \text{ factors}}$
2. $a^0 = 1$
3. $a^{-n} = \dfrac{1}{a^n}$
4. $a^{1/n} = \sqrt[n]{a}$

   $a^{m/n} = \sqrt[n]{a^m} = \left(\sqrt[n]{a}\right)^m$ $\quad$ m is any integer

11 Laws of Exponents Let a and b be positive numbers and let r and s be any rational numbers (that is, ratios of integers). Then

1. $a^r \times a^s = a^{r+s}$
2. $\dfrac{a^r}{a^s} = a^{r-s}$
3. $(a^r)^s = a^{rs}$
4. $(ab)^r = a^r b^r$
5. $\left(\dfrac{a}{b}\right)^r = \dfrac{a^r}{b^r}$ $\quad b \neq 0$

In words, these five laws can be stated as follows:

1. To multiply two powers of the same number, we add the exponents.
2. To divide two powers of the same number, we subtract the exponents.
3. To raise a power to a new power, we multiply the exponents.
4. To raise a product to a power, we raise each factor to the power.
5. To raise a quotient to a power, we raise both numerator and denominator to the power.

EXAMPLE 17

(a) $2^8 \times 8^2 = 2^8 \times (2^3)^2 = 2^8 \times 2^6 = 2^{14}$

(b) $$\frac{x^{-2} - y^{-2}}{x^{-1} + y^{-1}} = \frac{\dfrac{1}{x^2} - \dfrac{1}{y^2}}{\dfrac{1}{x} + \dfrac{1}{y}} = \frac{\dfrac{y^2 - x^2}{x^2y^2}}{\dfrac{y + x}{xy}} = \frac{y^2 - x^2}{x^2y^2} \cdot \frac{xy}{y + x}$$
$$= \frac{(y - x)(y + x)}{xy(y + x)} = \frac{y - x}{xy}$$

(c) $4^{3/2} = \sqrt{4^3} = \sqrt{64} = 8$ Alternative solution: $4^{3/2} = \left(\sqrt{4}\right)^3 = 2^3 = 8$

(d) $\dfrac{1}{\sqrt[3]{x^4}} = \dfrac{1}{x^{4/3}} = x^{-4/3}$

(e) $\left(\dfrac{x}{y}\right)^3 \left(\dfrac{y^2x}{z}\right)^4 = \dfrac{x^3}{y^3} \cdot \dfrac{y^8x^4}{z^4} = x^7y^5z^{-4}$ ■

Exercises

A Click here for answers.

1–16 ■ Expand and simplify.

1. $(-6ab)(0.5ac)$
2. $-(2x^2y)(-xy^4)$
3. $2x(x - 5)$
4. $(4 - 3x)x$
5. $-2(4 - 3a)$
6. $8 - (4 + x)$
7. $4(x^2 - x + 2) - 5(x^2 - 2x + 1)$
8. $5(3t - 4) - (t^2 + 2) - 2t(t - 3)$
9. $(4x - 1)(3x + 7)$
10. $x(x - 1)(x + 2)$
11. $(2x - 1)^2$
12. $(2 + 3x)^2$
13. $y^4(6 - y)(5 + y)$
14. $(t - 5)^2 - 2(t + 3)(8t - 1)$
15. $(1 + 2x)(x^2 - 3x + 1)$
16. $(1 + x - x^2)^2$

17–28 ■ Perform the indicated operations and simplify.

17. $\dfrac{2 + 8x}{2}$
18. $\dfrac{9b - 6}{3b}$
19. $\dfrac{1}{x + 5} + \dfrac{2}{x - 3}$
20. $\dfrac{1}{x + 1} + \dfrac{1}{x - 1}$
21. $u + 1 + \dfrac{u}{u + 1}$
22. $\dfrac{2}{a^2} - \dfrac{3}{ab} + \dfrac{4}{b^2}$
23. $\dfrac{x/y}{z}$
24. $\dfrac{x}{y/z}$
25. $\left(\dfrac{-2r}{s}\right)\left(\dfrac{s^2}{-6t}\right)$
26. $\dfrac{a}{bc} \div \dfrac{b}{ac}$
27. $\dfrac{1 + \dfrac{1}{c - 1}}{1 - \dfrac{1}{c - 1}}$
28. $1 + \dfrac{1}{1 + \dfrac{1}{1 + x}}$

29–48 ■ Factor the expression.

29. $2x + 12x^3$
30. $5ab - 8abc$
31. $x^2 + 7x + 6$
32. $x^2 - x - 6$
33. $x^2 - 2x - 8$
34. $2x^2 + 7x - 4$
35. $9x^2 - 36$
36. $8x^2 + 10x + 3$
37. $6x^2 - 5x - 6$
38. $x^2 + 10x + 25$
39. $t^3 + 1$
40. $4t^2 - 9s^2$
41. $4t^2 - 12t + 9$
42. $x^3 - 27$
43. $x^3 + 2x^2 + x$
44. $x^3 - 4x^2 + 5x - 2$
45. $x^3 + 3x^2 - x - 3$
46. $x^3 - 2x^2 - 23x + 60$
47. $x^3 + 5x^2 - 2x - 24$
48. $x^3 - 3x^2 - 4x + 12$

49–54 ■ Simplify the expression.

49. $\dfrac{x^2 + x - 2}{x^2 - 3x + 2}$
50. $\dfrac{2x^2 - 3x - 2}{x^2 - 4}$
51. $\dfrac{x^2 - 1}{x^2 - 9x + 8}$
52. $\dfrac{x^3 + 5x^2 + 6x}{x^2 - x - 12}$
53. $\dfrac{1}{x + 3} + \dfrac{1}{x^2 - 9}$

54. $\dfrac{x}{x^2 + x - 2} - \dfrac{2}{x^2 - 5x + 4}$

55–60 ■ Complete the square.

55. $x^2 + 2x + 5$ **56.** $x^2 - 16x + 80$

57. $x^2 - 5x + 10$ **58.** $x^2 + 3x + 1$

59. $4x^2 + 4x - 2$ **60.** $3x^2 - 24x + 50$

61–68 ■ Solve the equation.

61. $x^2 + 9x - 10 = 0$ **62.** $x^2 - 2x - 8 = 0$

63. $x^2 + 9x - 1 = 0$ **64.** $x^2 - 2x - 7 = 0$

65. $3x^2 + 5x + 1 = 0$ **66.** $2x^2 + 7x + 2 = 0$

67. $x^3 - 2x + 1 = 0$ **68.** $x^3 + 3x^2 + x - 1 = 0$

69–72 ■ Which of the quadratics are irreducible?

69. $2x^2 + 3x + 4$ **70.** $2x^2 + 9x + 4$

71. $3x^2 + x - 6$ **72.** $x^2 + 3x + 6$

73–76 ■ Use the Binomial Theorem to expand the expression.

73. $(a + b)^6$ **74.** $(a + b)^7$

75. $(x^2 - 1)^4$ **76.** $(3 + x^2)^5$

77–82 ■ Simplify the radicals.

77. $\sqrt{32}\,\sqrt{2}$ **78.** $\dfrac{\sqrt[3]{-2}}{\sqrt[3]{54}}$ **79.** $\dfrac{\sqrt[4]{32x^4}}{\sqrt[4]{2}}$

80. $\sqrt{xy}\,\sqrt{x^3y}$ **81.** $\sqrt{16a^4b^3}$ **82.** $\dfrac{\sqrt[5]{96a^6}}{\sqrt[5]{3a}}$

83–100 ■ Use the Laws of Exponents to rewrite and simplify the expression.

83. $3^{10} \times 9^8$ **84.** $2^{16} \times 4^{10} \times 16^6$

85. $\dfrac{x^9(2x)^4}{x^3}$ **86.** $\dfrac{a^n \times a^{2n+1}}{a^{n-2}}$

87. $\dfrac{a^{-3}b^4}{a^{-5}b^5}$ **88.** $\dfrac{x^{-1} + y^{-1}}{(x + y)^{-1}}$

89. $3^{-1/2}$ **90.** $96^{1/5}$

91. $125^{2/3}$ **92.** $64^{-4/3}$

93. $(2x^2y^4)^{3/2}$ **94.** $(x^{-5}y^3z^{10})^{-3/5}$

95. $\sqrt[5]{y^6}$ **96.** $(\sqrt[4]{a})^3$

97. $\dfrac{1}{(\sqrt{t})^5}$ **98.** $\dfrac{\sqrt[8]{x^5}}{\sqrt[4]{x^3}}$

99. $\sqrt[4]{\dfrac{t^{1/2}\sqrt{st}}{s^{2/3}}}$ **100.** $\sqrt[4]{r^{2n+1}} \times \sqrt[4]{r^{-1}}$

101–108 ■ Rationalize the expression.

101. $\dfrac{\sqrt{x} - 3}{x - 9}$ **102.** $\dfrac{(1/\sqrt{x}) - 1}{x - 1}$

103. $\dfrac{x\sqrt{x} - 8}{x - 4}$ **104.** $\dfrac{\sqrt{2 + h} + \sqrt{2 - h}}{h}$

105. $\dfrac{2}{3 - \sqrt{5}}$ **106.** $\dfrac{1}{\sqrt{x} - \sqrt{y}}$

107. $\sqrt{x^2 + 3x + 4} - x$ **108.** $\sqrt{x^2 + x} - \sqrt{x^2 - x}$

109–116 ■ State whether or not the equation is true for all values of the variable.

109. $\sqrt{x^2} = x$ **110.** $\sqrt{x^2 + 4} = |x| + 2$

111. $\dfrac{16 + a}{16} = 1 + \dfrac{a}{16}$ **112.** $\dfrac{1}{x^{-1} + y^{-1}} = x + y$

113. $\dfrac{x}{x + y} = \dfrac{1}{1 + y}$ **114.** $\dfrac{2}{4 + x} = \dfrac{1}{2} + \dfrac{2}{x}$

115. $(x^3)^4 = x^7$

116. $6 - 4(x + a) = 6 - 4x - 4a$

Answers

1. $-3a^2bc$ **2.** $2x^3y^5$ **3.** $2x^2 - 10x$ **4.** $4x - 3x^2$
5. $-8 + 6a$ **6.** $4 - x$ **7.** $-x^2 + 6x + 3$
8. $-3t^2 + 21t - 22$ **9.** $12x^2 + 25x - 7$
10. $x^3 + x^2 - 2x$ **11.** $4x^2 - 4x + 1$
12. $9x^2 + 12x + 4$ **13.** $30y^4 + y^5 - y^6$
14. $-15t^2 - 56t + 31$ **15.** $2x^3 - 5x^2 - x + 1$
16. $x^4 - 2x^3 - x^2 + 2x + 1$ **17.** $1 + 4x$ **18.** $3 - 2/b$
19. $\dfrac{3x + 7}{x^2 + 2x - 15}$ **20.** $\dfrac{2x}{x^2 - 1}$ **21.** $\dfrac{u^2 + 3u + 1}{u + 1}$
22. $\dfrac{2b^2 - 3ab + 4a^2}{a^2b^2}$ **23.** $\dfrac{x}{yz}$ **24.** $\dfrac{zx}{y}$ **25.** $\dfrac{rs}{3t}$
26. $\dfrac{a^2}{b^2}$ **27.** $\dfrac{c}{c - 2}$ **28.** $\dfrac{3 + 2x}{2 + x}$ **29.** $2x(1 + 6x^2)$
30. $ab(5 - 8c)$ **31.** $(x + 6)(x + 1)$ **32.** $(x - 3)(x + 2)$
33. $(x - 4)(x + 2)$ **34.** $(2x - 1)(x + 4)$
35. $9(x - 2)(x + 2)$ **36.** $(4x + 3)(2x + 1)$
37. $(3x + 2)(2x - 3)$ **38.** $(x + 5)^2$
39. $(t + 1)(t^2 - t + 1)$ **40.** $(2t - 3s)(2t + 3s)$
41. $(2t - 3)^2$ **42.** $(x - 3)(x^2 + 3x + 9)$
43. $x(x + 1)^2$ **44.** $(x - 1)^2(x - 2)$
45. $(x - 1)(x + 1)(x + 3)$ **46.** $(x - 3)(x + 5)(x - 4)$
47. $(x - 2)(x + 3)(x + 4)$ **48.** $(x - 2)(x - 3)(x + 2)$
49. $\dfrac{x + 2}{x - 2}$ **50.** $\dfrac{2x + 1}{x + 2}$ **51.** $\dfrac{x + 1}{x - 8}$ **52.** $\dfrac{x(x + 2)}{x - 4}$
53. $\dfrac{x - 2}{x^2 - 9}$ **54.** $\dfrac{x^2 - 6x - 4}{(x - 1)(x + 2)(x - 4)}$
55. $(x + 1)^2 + 4$ **56.** $(x - 8)^2 + 16$ **57.** $\left(x - \frac{5}{2}\right)^2 + \frac{15}{4}$
58. $\left(x + \frac{3}{2}\right)^2 - \frac{5}{4}$ **59.** $(2x + 1)^2 - 3$
60. $3(x - 4)^2 + 2$ **61.** $1, -10$ **62.** $-2, 4$

63. $\dfrac{-9 \pm \sqrt{85}}{2}$ **64.** $1 \pm 2\sqrt{2}$ **65.** $\dfrac{-5 \pm \sqrt{13}}{6}$
66. $\dfrac{-7 \pm \sqrt{33}}{4}$ **67.** $1, \dfrac{-1 \pm \sqrt{5}}{2}$ **68.** $-1, -1 \pm \sqrt{2}$
69. Irreducible **70.** Not irreducible
71. Not irreducible (two real roots) **72.** Irreducible
73. $a^6 + 6a^5b + 15a^4b^2 + 20a^3b^3 + 15a^2b^4 + 6ab^5 + b^6$
74. $a^7 + 7a^6b + 21a^5b^2 + 35a^4b^3 + 35a^3b^4 + 21a^2b^5 + 7ab^6 + b^7$
75. $x^8 - 4x^6 + 6x^4 - 4x^2 + 1$
76. $243 + 405x^2 + 270x^4 + 90x^6 + 15x^8 + x^{10}$
77. 8 **78.** $-\frac{1}{3}$ **79.** $2|x|$ **80.** $x^2|y|$
81. $4a^2b\sqrt{b}$ **82.** $2a$ **83.** 3^{26} **84.** 2^{60} **85.** $16x^{10}$
86. a^{2n+3} **87.** $\dfrac{a^2}{b}$ **88.** $\dfrac{(x + y)^2}{xy}$ **89.** $\dfrac{1}{\sqrt{3}}$
90. $2^5\sqrt{3}$ **91.** 25 **92.** $\frac{1}{256}$ **93.** $2\sqrt{2}|x|^3y^6$
94. $\dfrac{x^3}{y^{9/5}z^6}$ **95.** $y^{6/5}$ **96.** $a^{3/4}$ **97.** $t^{-5/2}$ **98.** $\dfrac{1}{x^{1/8}}$
99. $\dfrac{t^{1/4}}{s^{1/24}}$ **100.** $r^{n/2}$ **101.** $\dfrac{1}{\sqrt{x} + 3}$ **102.** $\dfrac{-1}{\sqrt{x} + x}$
103. $\dfrac{x^2 + 4x + 16}{x\sqrt{x} + 8}$ **104.** $\dfrac{2}{\sqrt{2 + h} - \sqrt{2 - h}}$
105. $\dfrac{3 + \sqrt{5}}{2}$ **106.** $\dfrac{\sqrt{x} + \sqrt{y}}{x - y}$
107. $\dfrac{3x + 4}{\sqrt{x^2 + 3x + 4} + x}$ **108.** $\dfrac{2x}{\sqrt{x^2 + x} + \sqrt{x^2 - x}}$
109. False **110.** False **111.** True **112.** False
113. False **114.** False **115.** False **116.** True