

Name: _____

Panther ID: _____

Exam 1 - MAC2311

Fall 2012

General Directions: *Read the problems carefully and provide answers exactly to what is requested. Use complete sentences and use notation correctly. Incomprehensible work is worthless. I am grading the work, not just the answer. Don't rush, don't try to do too many steps of a computation at once; work carefully. Good luck!*

1. (10 pts) Consider the function $f(x) = \sqrt{3 - 2x}$.

(a) (4 pts) What is the domain of f ? What is the range of f ? Write your answers using interval notation.

(b) (6 pts) Find a formula for the inverse function $f^{-1}(x)$.

2. (10 pts) (a) (5 pts) Solve for x (leave your answer in logarithm form): $3e^{-10x} = 5$.

(b) (5 pts) Use the triangle method to find an alternative expression for $\sin(\arctan x)$.

3. (6 pts) A population of chipmunks moves into a new forest at time $t = 0$. At time t (in months), the population numbers $P(t)$ chipmunks. With one sentence express in words what each of the following tells you.

(a) $P(12) = 620$;

(b) $\frac{P(12) - P(0)}{12} = 50$;

(c) $P'(12) = -10$.

4. (12 pts) (a) (8 pts) Sketch the graph of a function $f(x)$ satisfying **all** of the following conditions.

(i) $f(0) = 0$;

(ii) The graph has a vertical asymptote at $x = 2$, and

$$\lim_{x \rightarrow 2^-} f(x) = +\infty, \quad \lim_{x \rightarrow 2^+} f(x) = -\infty;$$

(iii) $\lim_{x \rightarrow -\infty} f(x) = -3$ and $\lim_{x \rightarrow +\infty} f(x) = -3$;

(b) (4 pts) Write a possible formula for a function satisfying **all** the conditions from part (a).

5. (10 pts) Given the function below

$$g(x) = \begin{cases} x^2 - 3x & \text{if } x < 3 \\ 3 & \text{if } x = 3 \\ -x + k & \text{if } x > 3 \end{cases}$$

(a) (4 pts) Is there a value of the constant k which will make the function continuous? Justify your answer.

(b) (6 pts) Sketch the graph of the function $g(x)$ when $k = 6$. Label carefully the coordinates of important points.

6. (20 pts) Find the following limits, if they exist (4 points each)

(a) $\lim_{x \rightarrow 3} \frac{x^2 - x - 6}{3 - x}$

(b) $\lim_{x \rightarrow 5} \frac{|x - 5|}{x - 5}$

(c) $\lim_{t \rightarrow 0} \frac{\sin(5t) - \tan t}{2t}$

(d) $\lim_{h \rightarrow 0} \frac{\frac{1}{x+h} - \frac{1}{x}}{h}$

(e) What does the limit in part (d) represent? (This should give you another way to check your answer to (d).)

7. (10 pts) Use the (ϵ, δ) definition to show that $\lim_{x \rightarrow 2} (100x - 1) = 199$.

8. (12 pts) Find the derivative of each of the following functions (6 points each):

(a) $f(x) = 3x^4 - \frac{1}{2x^2} + 5$

(b) $g(x) = \frac{4 - x^2}{\sqrt{x}}$

9. (10 pts) (a) (4 pts) Fill in the blanks:

$$(f(x)g(x))' = \underline{\hspace{10cm}}$$

(b) (6 pts) Using part (a), find the rule for computing $(f(x)g(x))''$.

10. (10 pts) Find the equation of a line that passes through the origin and which is tangent to the curve $y = x^2 + 3$.