Worksheet/Take-home Quiz - 11/19/13 Calc. I, F'13 **Name:** ____________________

1. In each case, find the general antiderivative:

(a) $\int (\sec^2 x + \frac{3}{\sqrt{x}} - 5)\, dx$ (b) $\int \frac{x^2 - 3}{2x}\, dx$

2. Solve the following initial value problems:

(a) $\frac{dy}{dx} = \sqrt{x}(6+5x), \;\; y(1) = 10$ (b) $\frac{d^2y}{dt^2} = \cos t + \sin t, \;\; y(0) = 3, \; y'(0) = 4$

3. (This problems helps you find on your own the proof of MVT) Recall MVT: Assume that $f(x)$ is a continuous function on a closed interval $[a, b]$ and assume that f is differentiable for all $x \in (a, b)$. Then there exists (at least) a point $c \in (a, b)$ so that

$$f'(c) = \frac{f(b) - f(a)}{b - a} .$$

Do the following steps to prove MVT.

(a) Draw a picture to illustrate MVT geometrically.

(b) Write the equation of the secant line connecting the points $(a, f(a))$ and $(b, f(b))$.

(c) Find a formula for the function $h(x)$ which measures the vertical distance between the y coordinate of the point on the secant line and the y coordinate of the point on the graph of f for an arbitrary value $x \in [a, b]$.

(d) Show that the function $h(x)$ satisfies the assumptions of Rolle's theorem for $x \in [a, b]$.

(e) Write the conclusion of Rolle's theorem applied to h and show that it translates exactly in the conclusion of MVT for f.