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Worksheet week 6 - Logistic models Calculus I Fall 2013

1. (This problem is adapted from Briggs Calculus) Scientists often use the logistic growth function

$$P(t) = \frac{P_0 K}{P_0 + (K - P_0)e^{-r_0 t}}$$

to model population growth, where P_0 is the initial population at time $t = 0$, K is the *carrying capacity*, and r_0 is the base growth rate. The carrying capacity is a theoretical upper bound on the total population that the surrounding environment can support.

(a) Confirm that $P(0) = P_0$ and compute $\lim_{t\to+\infty} P(t)$.

(b) Compute $P'(t)$. Observe that if $P_0 < K$, then $P'(t) > 0$, so $P(t)$ will be an increasing function towards the limiting value. Sketch its graph.

(c) The population of the world reached 6 billion in 1999 ($t = 0$). Assuming Earth's carrying capacity is 15 billion and the base growth rate is $r_0 = 0.025$, write a logistic growth function for the world's population (in billions). According to this model, when will world's population reach 12 billion?

(d) The *relative growth rate* r of a function f measures the rate of change of a function compared with its value at a particular point. It is computed as $r(t) = f'(t)/f(t)$. For the problem in part (c), show that relative growth of the population at $t = 0$ was $r(0) = 0.015$. This means the worlds population was growing at 1.5% per year in 1999.

(e) Compute $r(10)$, $r(20)$ and then evaluate

$$\lim_{t\to+\infty} r(t) = \lim_{t\to+\infty} \frac{P'(t)}{P(t)}\ .$$

What does this say about the relative growth rate of the population in the long run?