

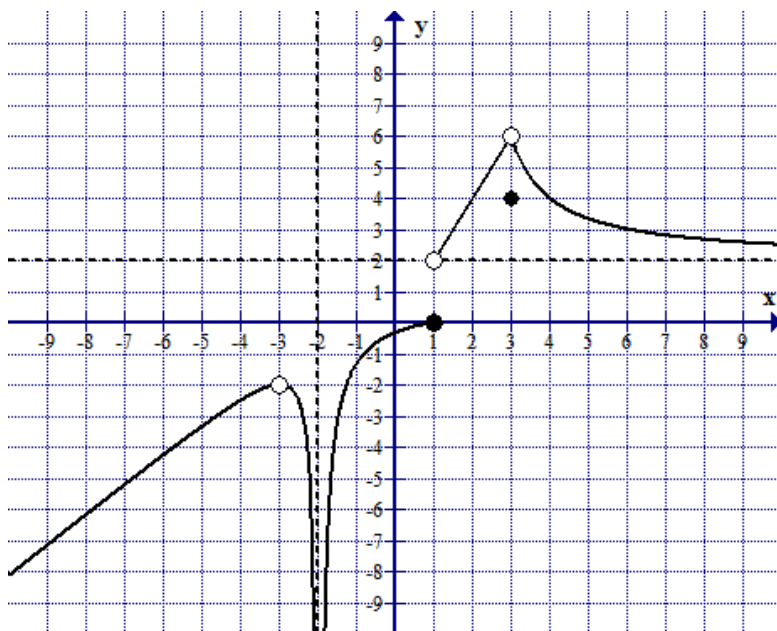
Name: _____

PID: _____

Fall 2016 -- MAC 2311- Exam 1

There are 7 problems for a total of 110 points. **Show your work**; an answer alone, even correct, will get no credit. An illegible solution will not be graded. Organize your work so it is clear what you do and why. **Calculators are not allowed.**

Problem 1.(15 pts) Use the graph of the function f given below to answer the questions that follow.



(i) (9 pts) Find the following limits (you don't have to show any work here)

a) $\lim_{x \rightarrow -\infty} f(x) =$

b) $\lim_{x \rightarrow +\infty} f(x) =$

c) $\lim_{x \rightarrow -2} f(x) =$

d) $\lim_{x \rightarrow 1^-} f(x) =$

e) $\lim_{x \rightarrow 1^+} f(x) =$

f) $\lim_{x \rightarrow 1} f(x) =$

g) $\lim_{x \rightarrow 4} f(x) =$

h) $\lim_{x \rightarrow 3} f(x) =$

i) $\lim_{x \rightarrow -3} f(x) =$

(ii) (4 pts) Is f continuous everywhere? If not, give x value(s) at which f has a discontinuity. Specify if any of the discontinuities is removable.

(iii) (2 pts) Is f continuous on the interval $[2,6]$? Answer and briefly justify.

Problem 2. (36 pts) Find the following limits. Show all work and explain clearly (6 pts each).

a) $\lim_{x \rightarrow 3^+} \frac{x}{3-x} =$

b) $\lim_{x \rightarrow -1} \frac{3x+3}{x^2+4x+3} =$

c) $\lim_{x \rightarrow 2} \frac{2x-4}{\sqrt{x+7}-3} =$

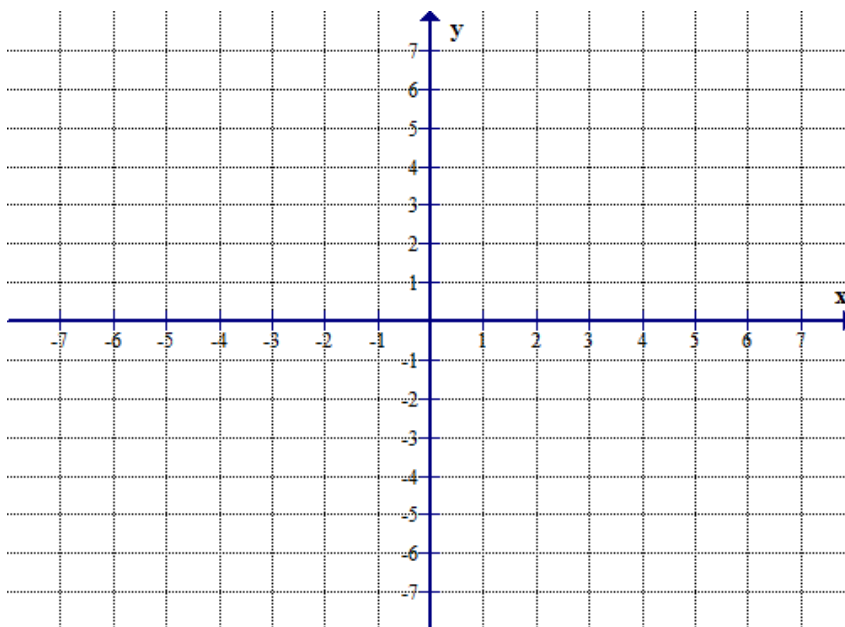
$$\text{d) } \lim_{x \rightarrow 1^+} \tan(\pi x) =$$

$$\text{e) } \lim_{x \rightarrow 0} \frac{\tan^2(4x)}{x \sin(5x)} =$$

$$\text{f) } \lim_{t \rightarrow -\infty} \frac{\sqrt{1 + 2t^2}}{2 + 3t} =$$

Problem 3. (12 pts) Sketch the graph of a function f satisfying ALL of the following properties:

- a) The domain of f is the set of all real numbers except $x = 2$;
- b) The function is continuous at all points except $x = -2$ and $x = 2$;
- c) $\lim_{x \rightarrow -2} f(x) = -\infty$, $f(-2) = 3$;
- d) $\lim_{x \rightarrow 2^-} f(x) = 0$, $\lim_{x \rightarrow 2^+} f(x) = 2$;
- e) $\lim_{x \rightarrow -\infty} f(x) = 0$;
- f) $\lim_{x \rightarrow +\infty} f(x) = 4$.



Problem 4. (10 pts) (a) (3 pts) Write the general (ϵ, δ) definition for

$$\lim_{x \rightarrow a} f(x) = L .$$

(b) (7 pts) Use the epsilon-delta definition to prove $\lim_{x \rightarrow -3} 10x + 3 = -27$.

Problem 5. (15 pts) A stone is dropped from the top of a building. Its position $s(t)$ in feet above the ground after t seconds is given by $s(t) = 160 - 16t^2$.

(a) (3 pts) When does the stone reach the ground?

(b) (5 pts) Find the average velocity of the stone during the first two seconds of its fall.

(b) (7 pts) Use limits to find the instantaneous velocity of the object at 2 seconds.

Problem 6. (12 pts) (a) (6 pts) Use the Intermediate Value Theorem to show that the equation $x^3 = 9x - 1$ has a solution in the interval $[0,1]$. Explain thoroughly.

(b) (6 pts) Use the Intermediate Value Theorem to show that the equation $x^3 = 9x - 1$ has three distinct real solutions. Explain thoroughly.

Problem 7. (10 pts) Choose ONE of the following:

(a) Assuming the inequality $\sin x \leq x \leq \tan x$ for any $x \in \left[0, \frac{\pi}{2}\right)$, prove that $\lim_{x \rightarrow 0^+} \frac{\sin x}{x} = 1$.

(b) Find the limit $\lim_{x \rightarrow +\infty} \frac{\cos(3x)}{x}$ and justify your answer.