NAME: Solution Key **Panther ID:** ____________

Exam 2 - MAC 2311 Spring 2016

Important Rules:

1. Unless otherwise mentioned, to receive full credit you MUST SHOW ALL YOUR WORK. Answers which are not supported by work might receive no credit.

2. Please turn your cell phone off at the beginning of the exam and place it in your bag, NOT in your pocket.

3. No electronic devices (cell phones, calculators of any kind, etc.) should be used at any time during the examination. Notes, texts or formula sheets should NOT be used either. Concentrate on your own exam. Do not look at your neighbor's paper or try to communicate with your neighbor. Violations of any type of this rule will lead to a score of 0 on this exam.

4. Solutions should be concise and clearly written. Incomprehensible work is worthless.

1. (35 pts) Find dy/dx. You do not have to simplify (7 pts each):

(a) $y = \dfrac{3x^4}{4} + \sqrt{x} - \sqrt{5} \quad = \dfrac{3}{4}x^4 + x^{\frac{1}{2}} - \sqrt{5}$

$$\frac{dy}{dx} = 3x^3 + \frac{1}{2}x^{-\frac{1}{2}} = 3x^3 + \frac{1}{2\sqrt{x}}$$

(b) $y = \dfrac{\tan x}{1 + x^2}$

$$\frac{dy}{dx} = \left(\frac{\tan x}{1+x^2}\right)' = \frac{\sec^2 x \cdot (1+x^2) - (\tan x) \cdot 2x}{(1+x^2)^2}$$

(c) $y = \sin^3(x^2)$ $= \left(\sin(x^2)\right)^3$

$$\frac{dy}{dx} = 3\left(\sin(x^2)\right)^2 \cdot \left(\sin(x^2)\right)' = 3\sin^2(x^2) \cdot \cos(x^2) \cdot 2x$$

(d) $y = \arctan(x \ln x)$

$$\frac{dy}{dx} = \frac{1}{1+(x\ln x)^2} \cdot (x \cdot \ln x)' = \frac{1}{1+(x\ln x)^2} \cdot \left(\ln x + \cancel{x} \cdot \frac{1}{\cancel{x}}\right)$$

or $$\boxed{\frac{dy}{dx} = \frac{\ln x + 1}{1+(x\ln x)^2}}$$

(e) $y = x^{e^x}$ $\leftarrow$ logarithmic diff.

$$\ln y = \ln\left(x^{e^x}\right) = e^x \cdot \ln x$$

$$(\ln y)' = (e^x \cdot \ln x)'$$

$$\frac{1}{y} \cdot y' = e^x \cdot \ln x + e^x \cdot \frac{1}{x}$$

so $$\frac{dy}{dx} = y \cdot \left(e^x \ln x + \frac{e^x}{x}\right) = \boxed{x^{e^x} \cdot e^x\left(\ln x + \frac{1}{x}\right)}$$

2. (10 points) These are True or False questions. Circle your answer. No partial credit – 2 points each.

(a) If $y = f(x) \cdot g(x)$, then $dy/dx = f'(x) \cdot g'(x)$. **True** **False**

need to apply product rule

(b) If $y = f(u)$ and $u = g(x)$, then $dy/dx = f'(u) \cdot g'(x)$. **True** **False**

It's the chain rule

(c) A function $f(x)$ has a horizontal tangent line at $x = 2$ if $f'(2) = 0$. **True** **False**

(d) If a function $f(x)$ is continuous at $x = 2$ then it is differentiable at $x = 2$. **True** **False**

continuous does not imply differentiable (the other way, yes)

(e) If $y = \sin(x)$, then $y'' = -y$. **True** **False**

$y' = \cos x \quad y'' = -\sin x = -y$

3. (12 pts) Show that $y = e^{-x^2}$ is a solution for the differential equation $y'' + 2xy' + 2y = 0$.

$$y' = \left(e^{-x^2}\right)' = e^{-x^2} \cdot (-2x) = -2xe^{-x^2}$$

$$y'' = \left(-2xe^{-x^2}\right)' = -2e^{-x^2} - 2x\,e^{-x^2} \cdot (-2x)$$

or $$y'' = e^{-x^2}(4x^2 - 2)$$

Plugging these in,

$$y'' + 2xy' + 2y = e^{-x^2}(4x^2-2) + 2x \cdot \left(-2xe^{-x^2}\right) + 2e^{-x^2}$$

$$= e^{-x^2}\left[4x^2 - 2 - 4x^2 + 2\right] = 0$$

So $y = e^{-x^2}$ is indeed a solution for

$$y'' + 2xy' + 2y = 0$$

4. (12 pts) A tank which initially holds 8000 gallons of water is being drained. Let $V(t)$ be the amount of water which still remains in the tank after t minutes since the drainage began. In each case, using one sentence explain the practical meaning of the given information. Use also units in your sentence.

(a) $V(20) = 3000$

Meaning: At $t=20$ minutes there are 3000 gallons of water left in the tank.

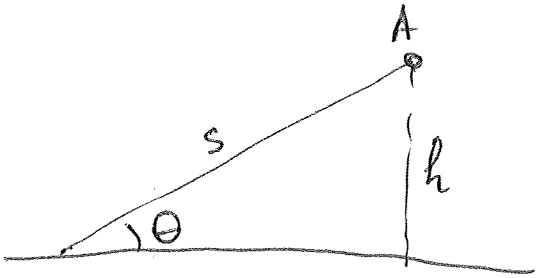
(b) $V'(20) = -100$

Meaning: At $t=20$ minutes the water is draining at a rate of 100 gals/minute

(c) $V(25) - V(30) = 350$

Meaning: In the time interval [25 min, 30 min], 350 gallons are drained from the tank.

5. (12 pts) An aircraft is climbing at a 30° angle to the horizontal. Determine how fast is the aircraft gaining altitude (in miles/minute) if its speed is 500 miles/hour?



$\theta = 30° = \frac{\pi}{6}$ ← constant

s and h are functions of time

Given $\frac{ds}{dt} = 500 \frac{mi}{h}$, find $\frac{dh}{dt} = ?$

$$\sin\frac{\pi}{6} = \frac{h}{s} \Rightarrow h = s \cdot \sin\left(\frac{\pi}{6}\right) = \frac{1}{2} s$$

so $$\frac{dh}{dt} = \frac{1}{2}\frac{ds}{dt} = \frac{1}{2}\cdot 500 = 250 \frac{mi}{h}$$

6. (12 pts) Find the equation of the tangent line to the curve $6x^3 + x^2y = y^3$ at the point $(1, 2)$.

Use implicit differentiation to find $\frac{dy}{dx}$.

$$(6x^3 + x^2y)' = (y^3)'$$

$$18x^2 + 2xy + x^2y' = 3y^2y'$$

$$18x^2 + 2xy = (3y^2 - x^2)y'$$

$$\text{so} \quad \frac{dy}{dx} = \frac{18x^2 + 2xy}{3y^2 - x^2}$$

$$m_{tan} = \left.\frac{dy}{dx}\right|_{(1,2)} = \frac{18 \cdot 1^2 + 2 \cdot 1 \cdot 2}{3 \cdot 2^2 - 1^2} = \frac{22}{11} = 2$$

So tangent line is

$\underline{y - 2 = 2 \cdot (x - 1)}$ or $\underline{y = 2x}$

7. (12 pts) Choose ONE:

(a) State and prove the formula for the derivative of a product of two functions.

(b) Find, with proof, the formula for $(\arccos x)'$.

see class notes or textbook