Name: ____________________________ **Panther ID:** ______________

Exam 1 Calculus II Spring 2013

To receive credit you MUST SHOW ALL YOUR WORK. Answers which are not supported by work will not be considered.

1. (12 pts) (a) A particle moves on a straight line and let $v(t)$ represent the velocity (in ft/second) of the particle at time t (in seconds), where $t \in [0, 10]$. Fill in the blanks with appropriate words.

Then $v'(t)$ represents ____ ,

while $\int_0^{10} |v(t)|\, dt$ represents ____ .

(b) Suppose that oil is leaking into the ocean from a damaged tanker at a rate of $r(t)$ gallons per day, where t is the time in days since the accident occurred. In one sentence, explain what the integral $\int_2^3 r(t)\, dt$ represents.

(a) Simplify as much as possible the expression

$$\frac{d}{dx}\left(\int_e^{e^x} (\ln t)^2\, dt\right)$$

2. (12 pts) (a) (6 pts) Find the area under the graph of $f(x) = \cos x$ over the interval $[0, \pi/2]$.

(b) (6 pts) On the interval $[0, \pi/2]$, the function $f(x) = \cos x$ has values between 0 and 1. How does the average value of the function $f(x) = \cos x$ on the interval $[0, \pi/2]$ compare with the value $\frac{1}{2}$? You may give either a computational or a geometric argument (or both).

3. (8 pts) A stone is dropped from the top of a building 192 ft tall. How long does it take until the stone reaches the ground? (Take the gravitational acceleration $g = 32$ ft/s^2.)

4. (28 pts) Compute each integral (7 pts each):

(a) $\displaystyle\int_1^4 \left(\frac{2}{x^2}+\frac{1}{\sqrt{x}}\right)\, dx\ dx$

(b) $\displaystyle\int_0^3 \sqrt{6x - x^2}$

(c) $\displaystyle\int_0^{\pi/4} 4\sin(2x)(1+\cos(2x))^3\, dx$

(d) $\displaystyle\int_{-\ln 2}^{\ln 2} \frac{e^x}{e^x+3}\, dx$

5. (8 pts) Use summation notation and then find the value of the sum: 2+4+6+8+ ... +998+1000

6. (24 pts) (a) (6 pts) Sketch the region bounded by the curves $y = \sqrt{x}$, $y = -2x + 10$, $y = 0$, and mark the coordinates of the three "corners" of the region. *Note:* This region is important for parts (b) and (c) of this problem. If you cannot do part (a), you may ask and get help from me, but you'll loose points for this part.

(b) (10 pts) Find the area of the region in part (a).
Computation is required for this part.

(c) (8 pts) The region in part (a) is now revolved around the line y-axis. Set up an integral (or integrals) that will give the volume of the solid obtained. The computation is **not required** for this part, just the set up of the integral is.

7. (14 pts) State and prove the second part of the Fundamental Theorem of Calculus (the one that involves the derivative of an integral). You may use without proof the Mean Value Theorem for Integrals, but indicate when you are using it.

8. (14 pts) The region enclosed by the semicircle $y = \sqrt{a^2 - x^2}$ and the x-axis is rotated around the x-axis. Find the volume of the solid obtained. Full computation is required. You should get a familiar formula. What is it?