Name: _____ **Panther ID:** _____

Exam 2 Calculus II Spring 2013

To receive credit you MUST SHOW ALL YOUR WORK.

1. (12 pts) Circle the correct answer (4 pts each):

(a) For the integral $\int \sqrt{x^2-3}\,dx$, the following substitution is helpful:

(i) $x = 3\tan\theta$ (ii) $x = \sqrt{3}\sin\theta$ (iii) $x = \sqrt{3}\sec\theta$ (iv) $3x = \sin\theta$ (v) $w = x^2 - 3$

(Don't spend time evaluating the integral. It is not required.)

(b) The partial fraction decomposition for $\frac{x+4}{x^2(x^2+4)^2}$ is of the form:

(i) $\dfrac{A}{x} + \dfrac{Bx+C}{x^2+4}$ (ii) $\dfrac{A}{x} + \dfrac{B}{x^2} + \dfrac{C}{x^2+4} + \dfrac{D}{(x^2+4)^2}$ (iii) $\dfrac{1}{x^5}$

(iv) $\dfrac{A}{x} + \dfrac{B}{x^2} + \dfrac{Cx+D}{x^2+4} + \dfrac{Ex+F}{(x^2+4)^2}$ (v) $\dfrac{A}{x^2} + \dfrac{Bx+C}{(x^2+4)^2}$

(c) The graph of the function $f(x)$ in the adjacent picture is rotated around the line $y = 1$ to generate a surface. The area of the surface obtained this way is given by:

(i) $\displaystyle\int_a^b \sqrt{1+(f'(x))^2}\,dx$ (ii) $\displaystyle\int_a^b 2\pi(f(x)-1)\sqrt{1+(f'(x))^2}\,dx$

(iii) $\displaystyle\int_a^b (f(x)-1)\,dx$ (iv) $\displaystyle\int_a^b 2\pi f(x)\sqrt{1+(f'(x))^2}\,dx$

2. (10 pts) State and prove the integration by parts formula.

3. (12 pts) The region enclosed by the curves $y = \frac{1}{x^2},\ y = 0,\ x = 1,\ x = 3$ is revolved about the y-axis. Compute the volume of the solid obtained. (Computation and picture are required for full credit.)

4. (12 pts) A hemispherical tank of radius 10 ft is located with its curved side underground and its flat side exactly at ground level. Assume that the tank is initially filled with gasoline of density $\rho = 45$ lb/ft^3. Set up the integral that represents the work required to empty the tank by a pump at ground level. (The calculation is not required, but a picture is.)

5. (40 pts) Compute each of the following. Full work required (10 pts each):

(*a*) $\displaystyle\int_0^{\pi/4} \sec^4 x \, dx$

(*b*) $\displaystyle\int e^x \sin 3x \, dx$

(c) $\displaystyle\int \frac{dx}{x^2\sqrt{4-x^2}}$

(d) $\displaystyle\int \frac{x}{x^4-1}\,dx$

6. (10 pts) If a thin rod has length L has constant density ρ, then the mass, M, of the rod is given by $M = \rho L$. Now suppose a thin rod occupies the interval $[a, b]$ on the x-axis, and suppose the density is non-constant: $\rho = \rho(x)$. Give a formula to compute the total mass of the rod in this case. Briefly explain your formula.

7. (14 pts) Find the arc length of the curve $x = t^3$, $y = t^2$, for $t \in [0, 1]$. The computation is required.