Name: Answer Key Panther ID: ____________

Exam 2 Calculus II Fall 2013

To receive credit you MUST SHOW ALL YOUR WORK.

1. (15 pts) Circle the correct answer (3 pts each):

(a) For the integral $\int \sqrt{9-x^2}\, dx$, the following substitution is helpful:

(i) $3x = \sec\theta$ (ii) $x = 3\sin\theta$ (iii) $w = 9 - x^2$ (iv) $x = 3\sec\theta$ (v) $x = 3\tan\theta$

(Don't spend time evaluating the integral. It is not required.)

(b) The partial fraction decomposition of $\dfrac{x+3}{x^4+9x^2}$ is of the form:

(i) $\dfrac{A}{x^2} + \dfrac{B}{x^2+9}$ (ii) $\dfrac{A}{x} + \dfrac{B}{x^2} + \dfrac{C}{x+3} + \dfrac{D}{(x+3)^2}$ (iii) $\dfrac{x+3}{x^4} + \dfrac{x+3}{9x^2}$

(iv) $\dfrac{A}{x} + \dfrac{B}{x^2} + \dfrac{Cx+D}{x^2+9}$ (v) $\dfrac{A}{x} + \dfrac{B}{x^2} + \dfrac{C}{x+3}$

(c) A function $f(x)$ is known to be continuous, positive and increasing when $x \in [-2, 2]$. Let L_4 be the left end-point approximation with 4 subdivisions of the integral $\int_{-2}^{2} f(x)\, dx$. Then compared with the integral, L_4 is an

(i) overestimate (ii) underestimate (iii) exact estimate (iv) cannot tell (more should be known about f)

(d) A function $g(x)$ is known to be continuous, positive and decreasing when $x \in [-2, 2]$. Let T_4 be the trapezoid approximation with 4 subdivisions of the integral $\int_{-2}^{2} f(x)\, dx$. Then compared with the integral, T_4 is an

(i) overestimate (ii) underestimate (iii) exact estimate (iv) cannot tell (more should be known about g)

(e) A function $h(x)$ is known to be continuous, positive and concave down when $x \in [-2, 2]$. Let M_4 be the midpoint approximation with 4 subdivisions of the integral $\int_{-2}^{2} h(x)\, dx$. Then compared with the integral, M_4 is an

(i) overestimate (ii) underestimate (iii) exact estimate (iv) cannot tell (more should be known about h)

2. (9 pts) Set up an integral that gives the arc-length of the curve $y = \ln x$, for $x \in [1, 2]$. Set up only, don't spend time trying to evaluate the integral.

$$S = \int_1^2 \sqrt{1 + \left(\frac{1}{x}\right)^2}\, dx$$

$$\boxed{= -\frac{1}{5}xe^{-5x} - \frac{1}{25}e^{-5x} + C}$$

3. (44 pts) Evaluate

(a) $\int xe^{-5x}\,dx = -\frac{1}{5}xe^{-5x} + \frac{1}{5}\int e^{-5x}\,dx$

I.B.P. $u = x \qquad dv = e^{-5x}\,dx$

$du = dx \qquad v = -\frac{1}{5}e^{-5x}$

(b) $\int_{-\infty}^{\infty} \frac{1}{1+x^2}\,dx = \pi$

note $\int \frac{1}{1+x^2}\,dx = \arctan x$ & $\lim_{x\to\pm\infty} \arctan x = \pm\frac{\pi}{2}$

(c) $\int \frac{1}{x^2-4x}\,dx = \boxed{\frac{1}{4}\left(\ln|x-4| - \ln|x|\right)}$

partial fractions $\frac{1}{x(x-4)} = \frac{1}{4}\left(\frac{1}{x-4} - \frac{1}{x}\right)$

(d) $\int \frac{1}{(x^2-4)^{3/2}}\,dx = \boxed{-\frac{x}{4\sqrt{x^2-4}} + C}$

start with sub $x = 2\sec\theta$

see class notes for work.

4. (14 pts) A boat is anchored so that the anchor is 200 ft below the surface. In water, the anchor weighs 1,500 lbs and the chain weighs 25 lbs/ft. How much work is required to raise the anchor to the surface? Computation is required.

$$W = \int_{y=0}^{y=200} (25y + 1500)\,dy = \ldots = 800{,}000 \text{ lbs}\cdot\text{ft}$$

5. (14 pts) Find the area of the region inside the cardioid $r = 1 - \sin\theta$ and above the x-axis. Full computation and a rough sketch are required.

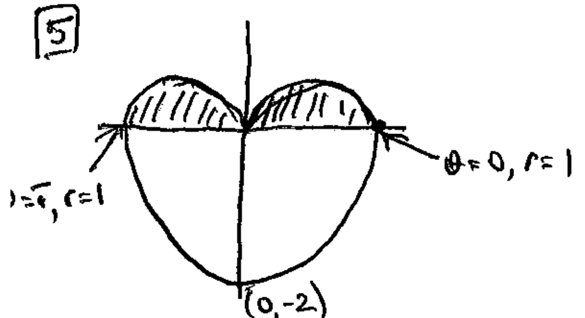
Between 6 or 7, choose one of them, clearly indicating your choice.

6. (14 pts) Derive the reduction formula for $\int \sin^n x\,dx$.

See class notes or solution manual

7. (14 pts) Derive the formula for surface area of a sphere of radius a, by rotating the semi-circle $x = a\cos t$, $y = a\sin t$, $t \in [0, \pi]$, around the x-axis. Full computation is required.

5



$$A = \frac{1}{2}\int_0^{\pi} r^2\,d\theta = 2\cdot\frac{1}{2}\int_0^{\frac{\pi}{2}} (1-\sin\theta)^2\,d\theta$$

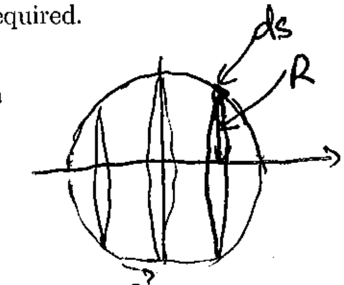
$$A = \int_0^{\frac{\pi}{2}} (1 - 2\sin\theta + \sin^2\theta)\,d\theta$$

$$A = \int_0^{\frac{\pi}{2}} \left(1 - 2\sin\theta + \frac{1-\cos(2\theta)}{2}\right) d\theta$$

$$A = \left(\frac{3}{2}\theta + 2\cos\theta - \frac{\sin(2\theta)}{4}\right)\Big|_{\theta=0}^{\frac{\pi}{2}}$$

$$\boxed{A = \frac{3\pi}{4} - 2}$$

7



$$S = \int_?^? 2\pi R\cdot ds$$

$$ds = \sqrt{(dx)^2 + (dy)^2} = \sqrt{(x')^2 + (y')^2}\,dt$$

$$ds = \sqrt{a^2\sin^2 t + a^2\cos^2 t}\,dt = a\,dt$$

$$R = y = a\sin t$$

$$S = \int_0^{\pi} 2\pi a\sin t\cdot a\,dt = 2\pi a^2(-\cos t)\Big|_{t=}^{t=}$$

$$\boxed{S = 4\pi a^2}$$