Name: _____   **Panther ID:** _____

FINAL EXAM   Calculus II   Spring 2013

Important Rules:

1. Unless otherwise mentioned, to receive full credit you MUST SHOW ALL YOUR WORK. Answers which are not supported by work might receive no credit.

2. Please turn your cell phone off at the beginning of the exam and place it in your bag, NOT in your pocket.

3. No electronic devices (cell phones, calculators of any kind, etc.) should be used at any time during the examination. Notes, texts or formula sheets should NOT be used either. Concentrate on your own exam. Do not look at your neighbor's paper or try to communicate with your neighbor. Violations of any type of this rule will lead to a score of 0 on this exam.

4. Solutions should be concise and clearly written. Incomprehensible work is worthless.

1. (12 pts) Circle the correct answer (work is not required for this problem) :

(a) The arc length of $y = \sin x$ from $x = 0$ to $x = \pi$ is given by

(i) $\dfrac{\sin \pi - \sin 0}{\pi - 0}$ (ii) $\displaystyle\int_0^\pi \sin x \, dx$ (iii) π (iv) $\displaystyle\int_0^\pi \sqrt{1 + \cos^2 x}\, dx$ (v) $\displaystyle\int_0^\pi \sqrt{1 + \sin^2 x}\, dx$

(b) The expression

$$\frac{d}{dx}\left(\int_1^{x^2} \frac{\sin t}{t}\, dt \right) \quad \text{is equivalent to}$$

(i) $\dfrac{\sin(x^2)}{x^2}$ (ii) $\sin 1$ (iii) $\dfrac{2\sin(x^2)}{x}$ (iv) $2x\dfrac{\sin t}{t}$ (v) 0

(c) The average value of the function $f(x)$ over the interval $[a,b]$ is

(i) $f(\dfrac{a+b}{2})$ (ii) $\dfrac{f(a)+f(b)}{2}$ (iii) $f'(b) - f'(a)$ (iv) $\dfrac{f(b) - f(a)}{b - a}$ (v) 0

(d) The partial fraction decomposition for $\frac{x+3}{x^4+9x^2}$ is of the form:

(i) $\dfrac{A}{x^2} + \dfrac{B}{x^2+9}$ (ii) $\dfrac{A}{x} + \dfrac{B}{x^2} + \dfrac{C}{x+3} + \dfrac{D}{(x+3)^2}$ (iii) $\dfrac{x+3}{x^4} + \dfrac{x+3}{9x^2}$

(iv) $\dfrac{A}{x} + \dfrac{B}{x^2} + \dfrac{Cx+D}{x^2+9}$ (v) $\dfrac{A}{x} + \dfrac{B}{x^2} + \dfrac{C}{x+3}$

2. (12 pts) The region bounded by the curve $y = \sqrt{x}$, the x-axis and the vertical line $x = 2$ is revolved around the line $x = 2$. Set up an integral to express the volume of the solid obtained. (Computation of the integral is NOT required, but the sketch of solid is.)

3. (40 pts) Evaluate (10 pts each)

(a) $\displaystyle\int x^3 \ln(x)\, dx$

(b) $\displaystyle\int \frac{1}{x^2 - 3x}\, dx$

(c) $\displaystyle\int_1^\infty x e^{-x^2}\, dx$

(d) $\displaystyle\int \frac{1}{x^2\sqrt{25 - x^2}}\, dx$

4. (6 pts) In each case, answer True or False. No work or justification is needed.

(a) If $0 < a_n < 1$ for all n, then the sequence $\{a_n\}$ is convergent.

(b) If $\sum_{k=1}^{\infty} a_k$ converges, then $a_k \to 0$.

(c) The series $1 - 1 + 1 - 1 + 1 - 1 + \ ...$ converges to 0.

5. (32 pts) Determine whether each of the following series converges conditionally, converges absolutely, or diverges. Be sure to state which test you are using and to show it applies to the series in question.

(a)(8 pts) $\displaystyle\sum_{k=1}^{\infty} \frac{k}{k^2 - 1}$

(b)(4 pts) $\displaystyle\sum_{k=1}^{\infty} \ln(k)$

(c)(12 pts) $\displaystyle\sum_{k=1}^{\infty} \frac{(-1)^k}{\sqrt{k+1}}$

(d)(8 pts) $\displaystyle\sum_{k=2}^{\infty} (-1)^k \pi^{-k}$

6. (14 pts) (a) (6 pts) Sketch the graph of the cardioid curve $r = 1 - \cos\theta$.

(b) (8 pts) Find the area of the region in the first quadrant that is inside the cardioid $r = 1 - \cos\theta$.

7. (20 pts) (a) (7 pts) Let n be a positive integer. Find, in terms of n, the area below the graph of $f(x) = 1/x$ and above the x-axis on the interval $[1, n]$.

(b) (7 pts) Write the left-hand point Riemann sum approximation of the area in part (a) corresponding to the division of the interval $[1, n]$ into sub-intervals of length 1 (your answer should use the summation notation). Is this approximation an over or an under estimate of the area in part (a)? (A picture is required.)

(c) (6 pts) Use parts (a) and (b) to show that the harmonic series is divergent.

8. (16 pts) Find the interval of convergence of the series

$$\sum_{k=1}^{\infty} \frac{(-2)^k x^{3k}}{k^{3/2}} \ .$$ Determine whether any of the endpoints belong to the interval of convergence.

9. (14 pts) Use the MacLaurin series for e^x to find the MacLaurin series for $F(x) = \int e^{-x^2} \, dx$. Assume $F(0) = 0$.