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## Worksheet 6 - MAC 2312, Fall 2013

1. Compute

(a) 
$$\int \frac{x^3 + 2x^2 + 1}{x^4 + x^2} dx$$

(b) 
$$\int_0^1 \frac{x^3 - 4x - 10}{x^2 - x - 6} dx$$

2. If a thin rod of length  $L$  (measured in  $cm$ ) has constant density  $\delta$  (measured in  $g/cm$ ), then the mass of the rod (in  $g$ ) is computed by  $m = \delta \cdot L$ . Next you are asked to derive a formula for the mass of a rod with variable density. Suppose that the rod is placed on the  $x$ -axis between the points  $x = a$  and  $x = b$ , and suppose the variable density at the point  $x$  in the rod is given by  $\delta(x)$ , for  $x \in [a, b]$ . Derive the formula for the total mass by dividing the rod in small pieces, etc.

3. A charged rod of length  $L$  is placed along the  $x$ -axis with the left side at the origin. It can be shown that the electric field produced by the rod at a point in the plane  $P(a, b)$  has horizontal, respectively, vertical components given by

$$E_x(P) = \int_{L-a}^{-a} \frac{\lambda x}{4\pi\epsilon_0(x^2 + b^2)^{3/2}} dx$$

$$E_y(P) = \int_{L-a}^{-a} \frac{\lambda b}{4\pi\epsilon_0(x^2 + b^2)^{3/2}} dx,$$

where  $\lambda$  is the charge density per unit length on the rod and  $\epsilon_0$  is the free space permittivity. (A picture will be shown on the board.) Evaluate the two integrals to determine simple formulas for the components of the electric field  $E_x(P), E_y(P)$ .

*Hint:* For one integral you need a trig substitution. The other can be done faster. Note also that  $\lambda$  and  $\epsilon_0$  are constants.