Name: ____________________ **SSN:** ____________________

Exam 1 MAC-2313 Fall 2017

To receive credit you MUST SHOW ALL YOUR WORK. Answers which are not supported by work will not be considered.

1. (20 pts) Given the vectors $\mathbf{u} = \mathbf{i} - \mathbf{k}$, $\mathbf{v} = -2\mathbf{i} + 2\mathbf{j} + \mathbf{k}$, $\mathbf{w} = 2\mathbf{j} - \mathbf{k}$, find each of the following:

(a) the orthogonal projection, $proj_{\mathbf{u}}\mathbf{v}$, of $\mathbf{v}$ on $\mathbf{u}$;

(b) the angle between $\mathbf{u}$ and $\mathbf{v}$;

(c) the volume of the parallelepiped determined by $\mathbf{u}$, $\mathbf{v}$ and $\mathbf{w}$.

(d) a vector $\mathbf{n}$ which is perpendicular to **all** three vectors $\mathbf{u}$, $\mathbf{v}$ and $\mathbf{w}$. Is this possible? Explain.

2. (8 pts) Circle the correct answer (2 pts each):

(a) The area of the parallelogram with adjacent sides the vectors $\mathbf{u}$ and $\mathbf{v}$ is given by:

(i) $\mathbf{u} \cdot \mathbf{v}$ (ii) $\|\mathbf{u}\| + \|\mathbf{v}\|$ (iii) $\mathbf{u} \times \mathbf{v}$ (iv) $\|\mathbf{u} \times \mathbf{v}\|$

(b) The vectors $\mathbf{u}$ and $\mathbf{v}$ are perpendicular when

(i) $\mathbf{u} \cdot \mathbf{v} = 0$ (ii) $|\mathbf{u}| = |\mathbf{v}|$ (iii) $\mathbf{u} = k\mathbf{v}$, for some constant k (iv) $\mathbf{u} \times \mathbf{v} = 0$.

(c) Let $\mathbf{r}(t)$ be a curve in 3-space (in an *arbitrary* parametrization). Denote by $\mathbf{T}(t)$, $\mathbf{N}(t)$, $\mathbf{B}(t)$ the unit tangent, normal, resp. binormal to the curve. The following two vectors are **always** collinear:

(i) $\mathbf{T}(t)$ and $\mathbf{r}'(t) \times \mathbf{r}''(t)$ (ii) $\mathbf{r}'(t)$ and $\mathbf{r}''(t)$ (iii) $\mathbf{B}(t)$ and $\mathbf{r}'(t) \times \mathbf{r}''(t)$ (iv) $\mathbf{N}(t)$ and $\mathbf{r}(t)$

(d) $\frac{d}{dt}(\mathbf{r}(t) \times \mathbf{r}'(t))$ is equivalent to:

(i) $\mathbf{r}(t) \times \mathbf{r}''(t)$ (ii) $\mathbf{r}'(t) + \mathbf{r}''(t)$ (iii) $\mathbf{r}'(t) \times \mathbf{r}''(t)$ (iv) $\|\mathbf{r}'(t)\|^2$

3. (12 pts) A bug moves in the plane so that at time t it's position vector is given by

$$\mathbf{r}(t) = \cos^3 t\,\mathbf{i} + \sin^3 t\,\mathbf{j}\,, \quad \text{for } 0 \le t \le \pi/2.$$

Find the total distance traveled by the bug in the time interval $0 \le t \le \pi/2$.

4. (18 pts) (a) (8 pts) Show that the line L_1 of intersection of the planes $x+2y-z=2$ and $3x+2y+2z=7$ is parallel to the line L_2 given by $x=1-6t$, $y=3+5t$, $z=2+4t$.

(b) (10 pts) Find the equation of the plane that contains both lines L_1, L_2 from part (a).

5. (20 pts) Consider the curve in 2-space $\mathbf{r}(t) = 3\cos t\,\mathbf{i} - \sqrt{3}\sin t\,\mathbf{j}$, $t \in [0, 2\pi]$.

(a) (6 pts) Sketch the curve in the xy-plane, indicating the direction of increasing t,

(b) (6 pts) Find the unit tangent $\mathbf{T}$ to the curve at point corresponding to $t = \pi/3$.

(c) (8 pts) Find the curvature $\kappa(t)$ of the curve.

6. (12 pts) Match the following equations with the appropriate surface:

(i) $x^2 + 2y^2 - 3z^2 = 1$

(ii) $x^2 + 2y^2 - 3z^2 = 0$

(iii) $(x + 1)^2 + 2(y - 1)^2 + 3(z - 2)^2 = 10$

(iv) $x + 2y^2 - 3z^2 = 1$

(v) $2y^2 - 3z^2 = 1$

(vi) $(x + 1)^2 - 2(y - 1)^2 - 3(z - 2)^2 = 10$.

(a) ellipsoid (b) hyperboloid with one sheet (c) hyperboloid with two sheets

(d) elliptic cone (e) hyperbolic paraboloid (f) hyperbolic cylinder

7. (10 pts) Find the point(s) of intersection (if any) of the line $x = 1 + t, y = 3 - t, z = 2t$ with the cylinder $x^2 + y^2 = 16$.

8. (12 pts) Choose ONE proof:
(A) Suppose that a particle is moving on a curve with constant speed. Show that at every moment the velocity vector is perpendicular to the acceleration vector.

(B) (Parametric equations of projectile motion) At the initial time $t = 0$ an object is launched from a height y_0 above the ground with an initial velocity vector $\mathbf{v}_0$ which makes an angle α with the horizontal. Starting from Newton's second law, derive the parametric equations of motion. (Assume that the gravitational force is the only force that acts on the object during the entire motion.)